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On superembedding approach to type IIB 7-branes

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Abstract: In search for a dynamical description of Q7-branes, which were known as solutions of supergravity equations and then conjectured to be dynamical objects of type IIB string theory, we study the superembedding description of 7-branes in curved type IIB supergravity superspace. With quite minimal and natural assumptions we have found that there is no place for Q7-branes as dynamical branes in superembedding approach. As Q7-brane was also considered as a bound state of two SD7-branes (this is to say of two 7-branes related to the D7-brane by different SL(2) transformation), our study might give implications for the old-standing problem of the covariant and supersymmetric description of multiple Dirichlet p-brane systems.

Keywords: p-branes, Superspaces, D-branes, M-Theory

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Contents

1 Introduction

Some times ago a new type of 7-brane solutions of type IIB supergravity was described in [1]. Latter this solutions were called Q-branes [2] and it was conjectured [2, 3] that they may correspond to a supersymmetric extended object, also called Q7-brane, in the same manner as the M2-brane solution of D=11 supergravity [4] is associated with the D=11 supermembrane action of [5]. A support for such a conjecture was the existence of a triplet of eight-form fields in D=10 type IIB supergravity, which had been found for the first time in [6], where the complete type IIB supergravity action has been build, and latter in the independent study of [7].

A candidate bosonic action for Q7-branes was considered in [3] (under the name of 'new seven-branes') and in [2]. Although some issues of supersymmetry have been discussed in [1] and in [2], neither the complete supersymmetric worldvolume action nor the complete supersymmetric set of equations of motion for the Q7-branes were proposed yet.

In this paper we explore the possibility to use the superembedding approach [8–19] (see [16, 17] for more references) to obtain a manifestly supersymmetric description of Q7-brane dynamics. The superembedding approach has already shown its usefulness for this type of problems. For instance, the equations of motion for the M-theory 5-brane (M5-brane) were obtained in [10] several months before the covariant action was constructed in [20] an, independently, in [21].

Furthermore, even if one is oriented on the covariant action, this also could be constructed on the basis of superembedding approach. We do not mean complete superfield action, like the so-called STV action for lower dimensional Brink-Schwarz superparticles [22].[1]

[1]STV is for Sorokin, Tkach and Volkov, the authors of the pioneer paper [22]. (See also [23, 24] for related studies). Such actions are known for superparticles and superstrings in superspaces with up to 16 supersymmetries, including the heterotic string without heterotic fermions [25], as well as for lower dimensional supermembranes (see [26, 27] and [16] for review and further references), but are not known neither for D=10 type II superbranes nor for D=11 M-branes. The problem appears also for heterotic string, on the stage when one tries to include heterotic fermions (or heterotic bosons). A number of approaches to superfield description of heterotic fermions were proposed [28, 29], but the most successful of them [29] is restricted to the case of SO(4) group, rather than SO(32) or $E_8 \otimes E_8$ charactersitic for the anomaly free heterotic string theory.

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We rather refer on the way of restoring the Green-Schwarz type action from the superembedding approach which was proposed in [30] (and can be treated as a bottom-up version of the generalized action principle for superbranes [31]).[2]

The analysis of possible closed 9-forms on type IIB target superspace (which we carry out in section 2 using the results of [19]) shows the existence of a candidate Wess-Zumino (WZ) term, provided one assumes the presence of two linearly independent gauge fields on the worldvolume.[3] Under independent gauge fields we understand such two worldvolume gauge potentials that their generalized field strengths involve different, linearly independent combinations of the pull-backs of the NS-NS (Nevieu-Schwarz – Nevieu-Schwarz) and RR (Ramond-Ramond) 2-form gauge potentials of the type IIB supergravity.[4] This WZ terms could also be used to construct the complete action on the line of [30] *provided there exists its complete worldvolume superspace extension*. This implies an existence of two linearly independent gauge supermultiplets in the 7-brane worldvolume superspace $W^{(8|16)}$ with 8-bosonic and 16 fermionic 'directions'.

Thus the search for a description of Q7-brane dynamics implies the study of the embedding of the 7-brane worldvolume superspace $W^{(8|16)}$ into the tangent superspace of type IIB supergravity $\mathcal{M}^{(10|16+16)}$, the simplest representative of which is the flat type IIB superspace $\Sigma^{(10|16+16)}$, and to find a possibility to describe supermultiplets with two independent worldvolume gauge field potentials. (We turn to this study in section 3).

With quite general assumption, including the standard form of the superembedding equation [8, 16, 22] and the gauge field constraints motivated by the consistency of the linear approximation, our results are negative: *there is no place for Q7-brane in the superembedding approach* (see section 4). In the light of universality of the supermebedding approach, this might be considered as an indication of *that Q7-brane is not a dynamical brane but rather a supersymmetric ground state of the system of two 'standard' 7-branes* (we call these *SD7-branes*, also the name of '(p,q)' seven branes can be used), each related to the *D7-brane* (Dirichlet super-7-brane) by different SL(2) transformation.[5]

Then, if supersymmetry is characteristic for the ground state only and the excited states of the system are not supersymmetric, the possible effective action for the two-brane system and the system of interacting equations of motion would not be κ-symmetric, while the ground state solution of the coupled equations of motion would possess supersymmetry. Such a picture was observed in [33] in an attempt to develop complete supersymmetric Lagrangian description for the interacting superstring — super-Dp-brane dynamical system. The conjecture suggested by our study is that the bosonic Q7-brane actions considered in [2, 3] are effective actions of this type, which do not allow a supersymmetric and κ-symmetric completion but do allow for a supersymmetric ground state solution.

[2] See [32] for an earlier approach to the action for heterotic string similar to the ones in [30, 31].

[3] The same conclusion, and also an expression for the (bosonic) Wess-Zumino term in terms of gauge potentials and pull-backs of supergravity fields can be found in [2].

[4] In other words, two independent worldvolume gauge fields are Goldstone fields for different, linearly independent combinations of the NS-NS and RR gauge transformations.

[5] In quantum theory $\mathrm{SL}(2,\mathbb{R})$ symmetry characteristic of type IIB supergravity is broken by Dirac quantization conditions for the brane charges down to its $\mathrm{SL}(2,\mathbb{Z})$ subgroup. In this paper we find convenient, following [19], to use the shortened notation SL(2).

In concluding section 5 we discuss our results and the arguments in favor of the above conjecture as well as their possible significance for understanding the general aspects of multi-brane interactions and, in particular, the old-standing problem of constructing the supersymmetric and Lorentz SO(1,9) symmetric Dp-brane action.[6]

To make the text lighter, we moved technical details to appendices A-E. Our notation and conventions are described in appendices A and B. We denote the bosonic and fermionic supervielbein forms of general type IIB supergravity superspace by

$$E^{\underline{A}} = (E^{\underline{a}}, \mathcal{E}^{\underline{\alpha}}) \; , \qquad \mathcal{E}^{\underline{\alpha}} := \mathcal{E}^{\alpha i} := (E^{\alpha 1} \; , \; E^{\alpha 2}) \; , \tag{1.1}$$

$\underline{a}, \underline{b}, \underline{c} = 0, 1, \ldots, 9$ are tangent space vector indices, $\alpha, \beta, \gamma = 1, \ldots, 16$ are $D = 10$ Majorana-Weyl spinor indices. The supervielbein obeys the type IIB supergravity constraints [40], the most important of which is $T_{\underline{\alpha\beta}}{}^{\underline{a}} = -2i\sigma^{\underline{a}}_{\underline{\alpha\beta}} := -2i\sigma^{\underline{a}}_{\alpha\beta}\delta_{ij}$, where $\sigma^{\underline{a}}_{\alpha\beta}$ are real symmetric 16×16 matrices (generalized Pauli matrices or Klebsh-Gordan coefficients for $\mathrm{SO}(1,9)$). Together with conventional constraints and their consequences this results in the following form of the type IIB superspace bosonic torsion 2-form, which can be loosely called *bosonic torsion constraint*,

$$T^{\underline{a}} := DE^{\underline{a}} = -i\mathcal{E} \wedge \underline{\sigma}^{\underline{a}}\mathcal{E} \;\; , \qquad \underline{\sigma}^{\underline{a}} := \sigma^{\underline{a}}_{\underline{\alpha\beta}} := \sigma^{\underline{a}}_{\alpha\beta}\delta_{ij} \; . \tag{1.2}$$

2 Problem statement. Candidate Wess-Zumino term for a hypothetical Q7-brane requires two worldvolume gauge fields.

Wess-Zumino (WZ) term describes coupling of a brane to antisymmetric tensor fields of the supergravity multiplets, as well as to axion and dilation (for D7-branes only axion contributes to the WZ term).

The candidate WZ term for Q7-brane was obtained in [2] from the requirement of gauge invariance. Although the bosonic construction of [2] allows for a straightforward supersymmetric generalization, we will present here its independent derivation, based on the results of [19]. This will allow us to see the relation between the D7-brane Wess-Zumino term and a triplet of nine forms which gives rise to the candidate WZ term for Q7, and will also demonstrate the strong necessity of the introduction of the second gauge field to write a closed covariant nine-form different from the D7-brane WZ term.

The coupling to the type IIB scalars (axion and dilaton) was studied in [2, 3] and the properties of such coupling were used intensively in the studies of these papers. However, if Q7-brane existed as a dynamical object, its action would make sense in any superspace background obeying the on-shell type IIB supergravity constraints (as it is for the case of

[6]This can be reformulated as a problem of lack of supersymmetry and Lorentz symmetry in the Myers action [34]. Notice that some progress in this direction was reached for the cases of low dimensions D, low dimensional branes and low co-dimensional branes [35–37]. Also a very interesting *minus one quantization* approach using *string with boundary fermions* was proposed in [38]; the quantization of the boundary fermions should reproduce the Myers action in this scheme (hence 'minus one quantization' name above). An attempt to reformulate the matrix diffeomorphism invariance as base-point-independence was discussed in [39] for bosonic D-branes.

the standard D7-brane action of [41]). In particular, it should exist the formulation in flat type IIB superspace, with vanishing axion and dilation.

It is natural to begin the study by considering this simplest case. The flat type IIB superspace $\Sigma^{(10|16+16)}$ can be characterized by the following superspace constraints (which differs from (1.2) by absence of the spin connection; $DE^{\underline{a}} = dE^{\underline{a}} - E^{\underline{b}} \wedge w_{\underline{b}}{}^{\underline{a}}$ in (1.2))

$$dE^{\underline{a}} = -i(E^{\alpha 1} \wedge E^{\beta 1} - E^{\alpha 2} \wedge E^{\beta 2})\sigma^{\underline{a}}_{\alpha\beta} = -i\mathcal{E} \wedge \underline{\sigma}^{\underline{a}}\mathcal{E} \,, \tag{2.1}$$

$$d\mathcal{E}^{\underline{\alpha}} = 0 \qquad \Leftrightarrow \qquad \begin{cases} dE^{\alpha 1} = 0 \,, \\ dE^{\alpha 2} = 0 \,. \end{cases} \tag{2.2}$$

This can be solved in terms of local coordinates by identifying supervielbein with the D=10 type IIB counterparts of the Volkov-Akulov supersymmetric 1-forms [42],

$$\begin{aligned} E^{\underline{a}} &= dX^{\underline{a}} - id\theta^1\sigma^{\underline{a}}\theta^1 - id\theta^2\sigma^{\underline{a}}\theta^2 = dX^{\underline{a}} - id\Theta\underline{\sigma}^{\underline{a}}\Theta \,, \\ \mathcal{E}^{\underline{\alpha}} &= d\Theta^{\underline{\alpha}} \qquad \Leftrightarrow \qquad \begin{cases} E^{\alpha 1} = d\theta^{\alpha 1} \,, \\ E^{\alpha 2} = d\theta^{\alpha 2} \,. \end{cases} \end{aligned} \tag{2.3}$$

2.1 Closed 9-forms in type IIB superspace

The Wess-Zumino term for a 7-brane in flat superspace can be characterized by a closed invariant 9-form (this can be identified with 9-cocycle of the Chevalley-Eilenberg cohomology [43, 44]). To find all the possible candidate 9-form WZ terms, one has to carry out the analysis of all the lower order invariant forms the wedge products of which can be used in the construction. Fortunately, we do not need to perform calculation ourselves as all such forms appear as flat superspace limit of the superspace field strengths of the SL(2) covariant formulation of the superspace type IIB supergravity elaborated in [19]. In addition to derivatives of axion and dilaton superfields, which are set to zero in the flat superspace limit ($F^{(1)}_R = 0$), the list of the field strength of SL(2) invariant formulation of type IIB supergravity [19] includes the doublet of three-forms, $F^{(3)}_R$, $R = 1, 2$, the singlet 5-form $F^{(5)}$, the doublet of seven-forms, $F^{(3)}_R$, and the triplet of the nine-forms $F^{(9)}_{RS} = F^{(9)}_{(RS)}$.[7] These obey the following Bianchi identities [19]

$$\begin{aligned} dF^{(3)}_R &= 0 \,, & dF^{(5)} &= -\epsilon^{RS} F^{(3)}_R \wedge F^{(3)}_S \,, \\ dF^{(7)}_R &= F^{(3)}_R \wedge F^{(5)} \,, & dF^{(9)}_{RS} &= F^{(3)}_{(R} \wedge F^{(7)}_{S)} \,, \end{aligned} \tag{2.4}$$

In flat type IIB superspace these field strengths are represented by the following supersymmetry invariant forms

$$F^{(3)}_R = -i\mathcal{E}^{\alpha i} \wedge \mathcal{E}^{\beta j} \wedge \bar{\sigma}^{(1)}_{\alpha\beta} (\tau_R)_{ij} \,, \tag{2.5}$$

$$F^{(5)} = i\mathcal{E}^{\alpha i} \wedge \mathcal{E}^{\beta j} \wedge \bar{\sigma}^{(3)}_{\alpha\beta}\, \epsilon_{ij} \,, \tag{2.6}$$

$$F^{(7)}_R = i\mathcal{E}^{\alpha i} \wedge \mathcal{E}^{\beta j} \wedge \bar{\sigma}^{(5)}_{\alpha\beta} (\epsilon_{RS}\tau^S)_{ij} \,, \tag{2.7}$$

$$F^{(9)}_{RS} = \frac{i}{2}\, \mathcal{E}^{\alpha i} \wedge \mathcal{E}^{\beta j} \wedge \bar{\sigma}^{(7)}_{\alpha\beta}\, \epsilon_{ij}\delta_{RS} \,, \tag{2.8}$$

[7] In this section we mainly use the SL(2) covariant notations of [19] (SL(2) symmetry reduces to SO(2) when axion and dilaton are set to zero, which is the case in flat superspace); below we give the relation to the NS-NS and R-R fields in a more familiar 'D-brane basis', which is not SL(2) covariant.

where

$$\bar{\sigma}^{(2n+1)}_{\alpha\beta} := \frac{1}{(2n+1)!} E^{a_{2n+1}} \wedge \ldots \wedge E^{a_1} \; \sigma_{a_1 \ldots a_{2n+1}\alpha\beta} \; , \tag{2.9}$$

and in flat superspace $(\tau_R)_{ij} = \delta^r_R (\tau_r)_{ij}$ with

$$(\tau_r)_{ij} = \frac{1}{\sqrt{2}} (\sigma_3 \, , \, \sigma_1) \ . \tag{2.10}$$

Clearly, these matrices are invariant under SO(2) but not under the SL(2) group; in general curved supergravity background $(\tau_R)_{ij} = \mathcal{U}_R{}^r (\tau_r)_{ij}$, where $\mathcal{U}_R{}^r$ is the axion-dilatom matrix providing the bridge between SO(2) and SL(2) groups.

This is the place to notice that the original papers of D-brane actions [41] used the NS-NS and RR field strengths H_3, H_7 and R_{2n+1} which obey the Bianchi identities (we set $R_1 = 0$, as it is in flat superspace)

$$dH_3 = 0 \; , \qquad dH_7 = -R_3 \wedge R_5 \; , \qquad dR_{2n+1} = -H_3 \wedge R_{2n-1} \; . \tag{2.11}$$

Comparing eqs. (2.5)–(2.8) with (2.11) one finds that the SL(2) multiplets of field strength can be expressed as

$$F^{(3)}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} H_3 \\ -R_3 \end{pmatrix} \; , \qquad F^{(5)} = -R_5 \; , \tag{2.12}$$

$$F^{(7)}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} R_7 \\ -H_7 \end{pmatrix} \; , \qquad F^{(9)}_{RS} = -\frac{1}{2} \delta_{RS} R_9 \; . \tag{2.13}$$

Notice that in our notation for the forms in 'D-brane basis', which we use in the main text, the number in subscript gives the order of form, while in the SL(2) covariant formalism, which we use only in this section, we keep the original notation of [19] in which the order of form is given in superscript by the number in brackets.

Now, to search for the candidate WZ term we need to use only Bianchi identities and eq. (2.12), (2.13). The explicit form of (2.5)–(2.8) is needed for calculating fermionic variations of the candidate action, but we will not turn to this issue in this paper.

Using the above field strengths one cannot construct any closed and supersymmetric invariant 9-form. This corresponds to non-existence of a super-7-brane without worldvolume gauge fields in type IIB superstring theory.

To write the WZ term of the D7-brane [41] one introduces the field strength $\mathcal{F}_2 = dA - \hat{B}_2$ of the worldvolume gauge field $A = d\xi^m A_m(\xi)$. These contains pull-back to the worldvolume $\hat{B}_2$ of the NS-NS two form gauge potential B_2, the field strength of which is $H_3 = dB_2$. Below in this section we will not distinguish pull-back from the form notationally i.e. we will omit hat symbols when this would not lead to a possible confusion. Thus

$$\mathcal{F}_2 = dA - B_2 \; , \qquad d\mathcal{F}_2 = -H_3 \; . \tag{2.14}$$

Completing the set of (pull-backs of) RR and NS-NS forms by the worldvolume field strength (2.14), one can find a closed invariant 9-form. It gives the (formal exterior derivative of the) D7 brane WZ term [41] and reads[8]

$$d\mathcal{L}_8^{WZ-D7} = R_9 + \mathcal{F}_2 \wedge R_7 + \frac{1}{2}\mathcal{F}_2 \wedge \mathcal{F}_2 \wedge R_5 + \frac{1}{3!}\mathcal{F}_2 \wedge \mathcal{F}_2 \wedge \mathcal{F}_2 \wedge R_3 \, . \tag{2.15}$$

This covariant 9-form is unique, as far as only one gauge field with the generalized field strength related to the NS-NS two form potential (2.14) is introduced.

To search for 9-form describing a candidate WZ term of the hypothetical Q7-brane action, first one has to introduce a doublet of the worldvolume field strengths involving two worldvolume gauge potentials and pull-backs of the doublet of two-form gauge field potentials (NS-NS and RR potentials),

$$\mathcal{F}_R^{(2)} := dA_R^{(1)} - C_R^{(2)} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathcal{F}_2 \\ -\tilde{\mathcal{F}}_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} dA - B_2 \\ -(d\tilde{A} - C_2) \end{pmatrix} \, , \tag{2.16}$$

so that

$$d\mathcal{F}_R^{(2)} := F_R^{(3)} \qquad \Leftrightarrow \qquad \begin{cases} H_3 = -d\mathcal{F}_2 \\ R_3 = -d\tilde{\mathcal{F}}_2 \end{cases} . \tag{2.17}$$

Now, starting from the triplet of the nine-form field strength, $F_{RS}^{(9)}$ (2.8),[9] and searching for an SL(2) covariant closed form $d\mathcal{L}_{RS}^{(8)}$, one finds the following triplet of closed 9-forms

$$d\mathcal{L}_{RS}^{(8)} = F_{RS}^{(9)} - \mathcal{F}_R^{(2)} \wedge F_S^{(7)} + \frac{1}{2}\mathcal{F}_R^{(2)} \wedge \mathcal{F}_S^{(2)} \wedge F_5 + \frac{1}{4}\mathcal{F}_R^{(2)} \wedge \mathcal{F}_S^{(2)} \wedge \epsilon^{R'S'}\, \mathcal{F}_{R'}^{(2)} \wedge \mathcal{F}_{S'}^{(2)} \, . \tag{2.18}$$

Notice that $(1,1)$ component of the triplet is similar, but not identical to the D7-brane WZ term (2.15). Namely

$$d\mathcal{L}_{R=1\,,S=1}^{(8)} = -\frac{1}{2}d\mathcal{L}_8^{WZ-D7} + \frac{1}{48}(\mathcal{F}_2 \wedge \mathcal{F}_2 \wedge \mathcal{F}_2 \wedge R_3 + 3\mathcal{F}_2 \wedge \mathcal{F}_2 \wedge \tilde{\mathcal{F}}_2 \wedge H_3) \, . \tag{2.19}$$

The difference is one of the representatives of the family of exact forms which appear in the presence of two worldvolume gauge fields,

$$\begin{aligned} \mathcal{C}_{2q+2p+5}^{(q,p)} &= (q+1)\mathcal{F}_2^{\wedge q} \wedge \tilde{\mathcal{F}}_2^{\wedge(p+1)} \wedge H_3 + (p+1)\mathcal{F}_2^{\wedge(q+1)} \wedge \tilde{\mathcal{F}}_2^{\wedge p} \wedge R_3 \; = \\ &= -d\left(\mathcal{F}_2^{\wedge(q+1)} \wedge \tilde{\mathcal{F}}_2^{\wedge(p+1)}\right) . \end{aligned} \tag{2.20}$$

Here $\mathcal{F}_2^{\wedge q} = \underbrace{\mathcal{F}_2 \wedge \ldots \wedge \mathcal{F}_2}_{q}$, etc.

[8] We use the notation $d\mathcal{L}_8^{WZ-D7}$ as far as the nine form (2.15) is exact in de Rahm cohomology, but call it closed because it represents a nontrivial cocycle of the Chevalley-Eilenberg cohomology [43]. This is to say, the form $\mathcal{L}_8^{WZ-D7}$ does exist (hence, exactness in de Rahm cohomology), but it is not invariant under supersymmetry (hence $d\mathcal{L}_8^{WZ-D7}$ is not exact in the Chevalley-Eilenberg cohomology). We refer on [44] for more discussion and intriguing applications.

[9] One easily notices that this triplet of 9-forms over flat superspace has actually only one independent component. Although in the case of curved superspace the other two nonvanishing contributions appear (the purely bosonic form contributions are related to derivatives of axion and dilaton by dualities, and also δ_{ij} in (2.8) is replaced by a bilinear of the axion-dilaton matrix, see [19]) the fact of reduction of a triplet of 9-form $F_{RS}^{(9)}$ to a singlet in the flat superspace limit already should rise suspicions concerning the existence of Q7-branes as dynamical objects.

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2.2 SD7-branes, Q7-branes and multiple (S)D-brane systems

A typical representative of the family of hypothetical Q7-branes would be characterized by the Wess-Zumino term

$$d\mathcal{L}^{(8)WZ-Q7} = Q^{RS} d\mathcal{L}^{(8)}_{RS}\,, \tag{2.21}$$

with a constant symmetric 2×2 matrix Q^{RS} the components of which give us, generically, three integer charges characterizing this 7-brane. Generic case corresponds to non-degenerate matrix Q^{RS}, $\det(Q^{RS}) \neq 0$. According to [2], the cases of charges forming the Q-matrix with positive determinant correspond to the Q7-branes while matrices with negative determinants do not correspond to any brane[10] (we will comment on this latter).

When determinant of the charge matrix is zero, its rank is 1 (if nonvanishing) and, hence, it can be expressed in term of one SL(2) vector, $Q^{RS} = q^R q^S$. The D7-brane correspond to $q^R = \binom{1}{0}$ and $Q^{RS} = \delta_1^R \delta_1^S$. The general $Q^{RS} = q^R q^S$ with $q^R \neq (0,0)$ corresponds to branes related to the D7-brane by SL(2) transformations. We call these *SD7-branes*.

We should stress that, in the light of the SL(2) duality invariance of the type IIB theory, the choice of D7-brane among SD7-branes is purely conventional. The SL(2) covariant description of the SD7-branes has been constructed in [19]. This makes clear that the property to have Wess-Zumino term expressed through only one gauge field cannot be specific just for one D7-brane. Indeed, the same expression with $\mathcal{F}_2$ replaced by $\sqrt{2}\, q^R \mathcal{F}^{(2)}_R$ would serve for the Wess-Zumino term of the SD7-brane with the charge matrix $Q^{RS} = q^R q^S$.

To resume, the charge matrices of different 7-branes are described by the table

$$\begin{array}{ccc} \text{D7-brane} & \leftrightarrow & Q^{RS} = \delta_1^R \delta_1^S \\ \text{SD7-brane} & \leftrightarrow & Q^{RS} = q^R q^S \\ \text{Q7-brane} & \leftrightarrow & \det(Q^{RS}) > 0 \end{array} \tag{2.22}$$

The last line actually can be equivalently written in the form of

$$\text{Q7-brane} \quad \leftrightarrow \quad Q^{RS} = \pm (q_1^R q_1^S + q_2^R q_2^S) \tag{2.23}$$

with nonvanishing doublets of charges q_1^R and q_2^R. This suggests to consider *Q7-brane as a bound state of two SD7-branes* characterized by charges q_1^R and q_2^R, respectively. More precisely, this treatment corresponds to sign plus in (2.23), while in the case of minus sign one should rather speak about bound state of two anti-SD7-branes. Notes that the case with different signs for the first and second contributions, which would correspond to a system of SD7-brane and an anti-SD7-brane, are excluded by the conditions of having $\det Q > 0$ [2].

[10] Notice that here we prefer to deal with the symmetric Q-matrix, which in notation of [2] reads $Q = \begin{pmatrix} p & -r/2 \\ -r/2 & q \end{pmatrix}$. Its relation with the traceless matrix $\mathbb{Q} = \begin{pmatrix} r/2 & p \\ -q & -r/2 \end{pmatrix}$, mainly used in [2] to describe Q7-brane charges, is given by $Q = \mathbb{Q}\, i\tau_2$. Referring on three independent charges, Q7-branes are also called (p,q,r)-brane, and the condition $\det Q = \det \mathbb{Q} > 0$ implies $pq > \frac{r^2}{4}$.

Thus, when considering Q7 as an interacting system of two SD7-branes, the seemingly mysterious requirement of $\det Q > 0$ just corresponds to the well known fact that the supersymmetry is broken in the system including a brane and an anti-brane.

The possibility to be treated as a system composed of two SD7-branes does not prevent Q7-brane from being described by an effective action and, in this sense, from being a dynamical object. However, in this case one should expect, at least, that such a hypothetical effective action for a bound state of two SD7-branes does possess certain symmetries, including Lorentz invariance and supersymmetry. If this is not the case, in particular, if an effective action does not possesses supersymmetry, one may think that the Q7-brane solution [3] does not correspond to a dynamical brane but rather to a particular *ground* state of a system of two interacting SD7-branes which does possesses supersymmetry (in distinction to a generic states of this system). An analogy comes from the study of supergravity solutions describing intersecting plane branes [49, 53]; in general such solutions are not supersymmetric, but supersymmetry appears for certain angles (the complete supersymmetry characteristic for one brane in the case of coincidence, one half of the supersymmetry of the one-brane solution for the orthogonal intersection, etc.). We will be coming back to this point in the next sections and, particularly, in the concluding section 5.

To conclude this section, let us notice a similarity of the problem of searching for the action for Q7-brane with the problem of Lagrangian description of the multiple-Dp-brane systems, which becomes transparent after understanding the possible treatment of Q7-brane as a coupled system of two SD7-branes. The problem with Q7-brane action, which looks relatively simple in the light of do not expecting a non-Abelian structure, might hence provide new insights for the multiple-Dp-brane system.[11]

2.3 Superembedding-based method to search for hypothetical Q7-brane action. Problem statement.

The action for SD7-brane was constructed in [19] by the method proposed in [30] starting from the superembedding description [9, 12]. Roughly, this procedure can be split on the following stages. One i) finds the Wess-Zumino term, ii) lifts it to the maximal worldvolume superspace of the p-brane, *restricted by the superembedding equation* (see below) and then iii) uses this superspace form to search for the kinetic, (presumably) Dirac-Born-Infeld-like (DBI) part of the action in an algorithmic manner. The detailed description of the stage (iii) is not needed for our purposes here, it can be found in [19, 30]. The first stage (i) was the subject of our sections 2.1 and 2.2. The main subject of our study below will be the possibility to progress in the second stage (ii).

Of course, the DBI part may be also constructed by searching for the κ-symmetric completion of the Wess-Zumino term, but the superembedding approach based method of [19, 30] is more algorithmic and, hence, more conclusive in the case of negative result (which we will actually arrive at in the case of Q7-branes).

[11] Let us recall that the commonly used Myers action [34], which predicted the 'dielectric brane effect' and was obtained by using T-duality arguments, do not possess neither supersymmetry nor Lorentz symmetry; see also footnote 6.

Now, having in hand the candidate Wess-Zumino term for the Q7-brane, it is natural to apply this superembedding-based method in our search for a complete Q7-brane action. The first stage in this direction should be, as we commented above, to lift the candidate Wess-Zumino term to the complete worldvolume superspace (N=1, d=8 superspace in the case of type IIB 7-branes) subject to the superembedding equation (discussed below). However, *this inevitably implies that two gauge fields living on the hypothetical Q7-brane worldvolume, are lifted to the worldvolume superspace*.

Thus, the first question to ask is whether a 7-brane worldvolume superspace can carry two super-1-form gauge potentials which are essentially different in the sense of that their invariant field strengths contain pull-backs of the different (linear independent) combinations of the NS-NS and R-R two-form potentials.

This will be the main subject of section 4. But before turning to it, in the next section 3 we describe the general features of the superembedding approach, specifying it for type IIB 7-branes, and its application to obtain D7-brane equations of motion.

3 Superembedding approach to type IIB 7-branes and description of D7-brane dynamics

3.1 Superembedding equation for type IIB 7-branes

3.1.1 Worldvolume superspace, pull-backs of target superspace superforms and superembedding equation

To write the most general and universal form of the *superembedding equation* for a super-p-brane in D=10 type IIB supergravity background, let us first denote the $d = p+1 \leq 10$ local bosonic coordinates and 16 fermionic coordinates of the worldvolume superspace $W^{(p+1|16)}$ by $\zeta^{\mathcal{M}} = (\xi^m, \eta^{\check{q}})$. Then let us notice that the embedding of $W^{(p+1|16)}$ into the tangent type IIB superspace $\Sigma^{(10|32)}$ with coordinates $Z^{\underline{M}} = (x^{\underline{m}}\ ,\ \theta^{\check{\alpha}1}\ ,\ \theta^{\check{\alpha}2}\ )$ can be described parametrically by specifying the set of coordinate functions, the *worldvolume superfields* $\hat{Z}^{\underline{M}}(\zeta)$,

$$W^{(p+1|16)} \in \Sigma^{(10|32)}: \qquad \begin{aligned} Z^{\underline{M}} &= \hat{Z}^{\underline{M}}(\zeta)\ , \\ \zeta^{\mathcal{M}} &= (\xi^m\ ,\ \eta^{\check{q}})\ , \qquad \hat{Z}^{\underline{M}}(\zeta) = (\hat{x}^{\underline{m}}(\zeta)\ , \hat{\theta}^{\check{\alpha}1}(\zeta)\ ,\ \hat{\theta}^{\check{\alpha}2}(\zeta))\ , \end{aligned} \tag{3.1}$$

with $\underline{m} = 0, 1, \ldots, 9$, $\check{\alpha} = 1, \ldots 16$, $\check{q} = 1, \ldots 16$ and $m = 0, 1, \ldots 7$ in the case of type IIB 7-branes. The superembedding equation is imposed on these coordinate functions.

Denoting the supervielbein of the worldvolume superspace $W^{(8|16)}$ by

$$e^A = d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^A(\zeta) = (e^a\ ,\ e^q)\ , \qquad a = 0, 1, \ldots, 7\ , \qquad q = 1, \ldots, 16\ , \tag{3.2}$$

one can decompose the pull-back $\hat{E}^{\underline{A}} := E^{\underline{A}}(\hat{Z})$ of the supervielbein of the target type IIB superspace $E^{\underline{A}} = dZ^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(Z) = (E^{\underline{a}}, E^{\alpha 1}, E^{\alpha 2})$, eq. (1.1), on the basis (3.2). In general, such a decomposition reads

$$\hat{E}^{\underline{A}} := E^{\underline{A}}(\hat{Z}) = d\hat{Z}^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(\hat{Z}) = e^b \hat{E}_b^{\underline{A}} + e^q \hat{E}_q^{\underline{A}}\ , \tag{3.3}$$

where $\hat{E}_b^{\underline{A}} := e_b^{\mathcal{M}}\partial_{\mathcal{M}}\hat{Z}^{\underline{M}}E_{\underline{M}}{}^{\underline{A}}(\hat{Z})$ and $\hat{E}_q^{\underline{A}} := e_q^{\mathcal{M}}\partial_{\mathcal{M}}\hat{Z}^{\underline{M}}E_{\underline{M}}{}^{\underline{A}}(\hat{Z})$.

The superembedding equation states that the fermionic component of the pull-back of the bosonic supervielbein form vanishes,

$$\hat{E}_q{}^{\underline{a}} := \nabla_q\hat{Z}^{\underline{M}}E_{\underline{M}}{}^{\underline{a}}(\hat{Z}) = 0\ , \qquad \nabla_q := e_q^{\mathcal{M}}(\zeta)\partial_{\mathcal{M}}\ , \quad \zeta^{\mathcal{M}} = (\xi^m, \eta^{\check{q}})\ . \tag{3.4}$$

This superembedding equation was first obtained form the STV action for $D = 3, 4$ dimensional superparticle [22] and was used as a basis to develop superembedding approach for D=10 superstrings and D=11 supermembrane (M2-brane) in [8] (see [16] for more references). The superembedding equation for Dp-branes and M5-brane were used to derive their equations of motion, respectively in [9] and [10], before the complete covariant action was found, respectively in [41] and [8, 21].

3.1.2 Linearized superembedding equation in 'static gauge' and Goldstone superfields

To create some feeling of the superembedding equation, it is useful to consider its linearized version in flat target superspace (see [9] for the case of Dp-branes). In this approximation a 7-brane is described by one complex (two real) bosonic superfield(s) $\widetilde{X}^z(\zeta) = (\widetilde{\bar{X}}^{\bar{z}}(\zeta))^*$ and 16 pseudo-real fermionic superfields $W^q(\zeta) = \gamma^0_{qp}(W^p(\zeta))^*$ which obey

$$D^0_q\tilde{X}^z = -2i(\delta + i\gamma^9)_{qp}W^p\ , \qquad D^0_q\tilde{\bar{X}}^{\bar{z}} = -2i(\delta - i\gamma^9)_{qp}W^p\ \ . \tag{3.5}$$

Here $D^0_q = \frac{\partial}{\partial\eta^q} + 2i\theta^p\gamma^a_{pq}\partial_a$, $\ p, q = 1, \ldots, 16$ are pseudo-Majorana d=8 spinor indices, γ^a_{pq} are the $d = 8$ gamma matrices, $a = 0, 1, \ldots, 7$, and $\gamma^9 = \gamma^0\gamma^1\ldots\gamma^7$. See appendix A for further details.

To arrive at (3.5) one uses the worldvolume diffeomorphism symmetry to fix the so-called static gauge in which the coordinates of the worldvolume superspace are identified with 8 of 10 bosonic and 16 of 32 fermionic coordinate functions,

$$\xi^a = \hat{X}^a(\xi,\eta) - iW(\xi,\eta)\gamma^a\eta\ , \qquad \eta^q = \hat{\theta}^{1q}(\xi,\eta) := \hat{\theta}^{1\alpha}(\xi,\eta)\delta_\alpha{}^q\ . \tag{3.6}$$

The remaining coordinate superfunctions, $\hat{X}^8$, $\hat{X}^9$ and $\hat{\theta}^{2\alpha}$, are associated to the Goldstone superfields $\widetilde{X}^z(\zeta) \equiv \widetilde{X}^z(\xi,\eta)$ and $W^q(\zeta) \equiv W^q(\xi,\eta)$. In linearized approximation it is convenient to define these by

$$\widetilde{X}^z(\xi,\eta) := \hat{X}^8 + i\hat{X}^9 + iW(\delta + i\gamma^9)\eta \quad \text{ and } \quad W^q(\xi,\eta) := -\hat{\theta}^{2\alpha}\delta_\alpha{}^p\gamma^9_{pq} - \eta^q \tag{3.7}$$

(equivalent to $\theta^2 = (\theta^1 + W)\gamma^9 = (\eta + W)\gamma^9$). This choice results in the simple form (3.5) of the linearized superembedding equation which provides the superfield description of the $d = 8$, $\mathcal{N} = 1$ scalar supermultiplet.

3.1.3 Equivalent form of superembedding equation, induced worldvolume supervielbein and moving frame variables

In the discussion below it is convenient to use the worldvolume supervielbein induced by superembedding. This implies, in particular, that the bosonic supervielbein form e^a is expressed by

$$\hat{E}^a := \hat{E}^{\underline{b}}u_{\underline{b}}{}^a = e^a \tag{3.8}$$

in terms of the pull-back to $W^{(8|16)}$ of the bosonic type IIB supervielbein, $\hat{E}^{\underline{b}}$, and the *moving frame variables* $u_{\underline{b}}{}^{a}$. These are eight ortogonal and normalized vectors

$$u_{\underline{a}}{}^{a}\eta^{\underline{ab}}u_{\underline{b}}{}^{b} = \eta^{ab} , \qquad a,b = 0,1,\ldots,7 , \qquad \underline{a}\, , \underline{b} = 0,1,\ldots,9 . \tag{3.9}$$

In flat superspace $E^{\underline{b}}$ has the form of (2.3); in general it obeys the type IIB supergravity constraints, the most essential of which, $T_{\alpha i\ \beta j}{}^{\underline{b}} = -2i\delta_{ij}\sigma^{\underline{b}}_{\alpha\beta}$, is included in (1.2).

One can complete the above set of orthogonal and normalized vectors $u_{\underline{a}}^{\ a}$ $(a = 0,1,...,7)$ by two orthogonal to them and also normalized vectors $u_{\underline{a}}^{\ i} = (u_{\underline{a}}^{\ 8}, u_{\underline{a}}^{\ 9})$, $u_{\underline{a}}^{\ i}u^{\underline{a}\,a} = 0$, $u_{\underline{a}}^{\ i}u^{\underline{a}\,j} = -\delta^{ij}$. As far as $u_{\underline{a}}^{\ a}$ are assumed to be tangential to the worldvolume, eq. (3.8), and linearly independent, these two are orthogonal to it, so that, taking into account the superembedding equation (3.4), $E^{i} := E^{\underline{a}}u_{\underline{a}}^{\ i} = 0$. As it was first noticed in [8], this gives an equivalent representation of the superembedding equation (3.4). It is convenient to collect $u_{\underline{a}}^{\ i} = (u_{\underline{a}}^{\ 8}\, , \, u_{\underline{a}}^{\ 9})$ in two complex vectors

$$u_{\underline{a}}^{\ z} = u_{\underline{a}}^{\ 8} + iu_{\underline{a}}^{\ 9} = (\bar{u}_{\underline{a}}^{\ \bar{z}})^{*} , \qquad u_{\underline{a}}^{\ z}u^{\underline{a}\ b} = 0 , \qquad u_{\underline{a}}^{\ z}u^{\underline{a}\ z} = 0 , \qquad \bar{u}_{\underline{a}}^{\ \bar{z}}u^{\underline{a}\ z} = -2 , \tag{3.10}$$

so that the above equivalent form of the superembedding equation reads

$$\hat{E}^{z} := \hat{E}^{\underline{a}}u_{\underline{a}}{}^{z} = 0 , \qquad \hat{E}^{\bar{z}} := \hat{E}^{\underline{a}}\bar{u}_{\underline{a}}{}^{\bar{z}} = 0 . \tag{3.11}$$

Actually, the superembedding equation (3.4) and the conventional constraint (3.8) can be collected together in the expressions for the pull-back of the type IIB bosonic supervielbein form,

$$\hat{E}^{\underline{a}} := E^{\underline{a}}(\hat{Z}(\zeta)) = e^{b}u_{b}{}^{\underline{a}}(\zeta) , \qquad u^{\underline{a}b}u_{\underline{a}}{}^{c} = \eta^{bc} , \qquad \begin{cases} \underline{a} \ = \ 0,1,\ldots,9 , \\ b,c = 0,1,\ldots,7 . \end{cases} \tag{3.12}$$

This equation can also be obtained by substitution of the original form of the superembedding equation, eq. (3.4), into the general decomposition of eq. (3.3) with $A = \underline{a}$. Then the only information which is explicit in (3.12), in comparison to the previously described equation, is the orthogonality and normalization of the coefficient matrices, $\hat{E}_{a}^{\underline{a}} = u_{a}^{\underline{a}}$. This corresponds to the conventional constraints of choosing the bosonic worldvolume supervielbein to be induced by (super)embedding, eq. (3.8).

3.2 Fermionic supervielbein induced by superembedding and spinor moving frame

To specify the induced supervielbein (3.2) of the worldvolume superspace, we need, besides (3.8), to express the 16 fermionic forms e^{q} (carrying the pseudo-Majorana spinor representation of SO(1,7), $(e^{q})^{*} = \gamma^{0}_{qp}e^{p}$) in terms of pull-backs of 32 fermionic forms (1.1) of the target type IIB superspace, $\mathcal{E}^{\underline{\alpha}} = \mathcal{E}^{\alpha i} = (E^{\alpha 1}\, , \, E^{\alpha 2})$. These latter carry the spinor indices of the Majorana representation of Spin(1,9).

To write such a fermionic conventional constraints we need in a 'bridge' ([46]) between Spin(1,7) and Spin(1,9) groups, which is to say, in a matrix variable carrying one Spin(1,7) and one Spin(1,9) indices. Such a bridge is given by *spinor moving frame* matrix

$$V_{\alpha}{}^{q} \ \in \mathrm{Spin}(1,7) , \qquad (V_{\alpha}{}^{q})^{*} = \gamma^{0}_{qp}V_{\alpha}{}^{q} \tag{3.13}$$

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providing a square root of the moving frame variables in the sense of that

$$\begin{aligned} V_\alpha \gamma^a V_\beta := V_\alpha{}^q \gamma^a_{qp} V_\beta{}^p &= \sigma^{\underline{b}}_{\alpha\beta} u_{\underline{b}}{}^a \, , \\ V_\alpha{}^q (\delta + i\gamma^9)_{qp} V_\beta{}^p &= \sigma^{\underline{b}}_{\alpha\beta} u_{\underline{b}}{}^z \, , \\ V_\alpha{}^q (\delta - i\gamma^9)_{qp} V_\beta{}^p &= \sigma^{\underline{b}}_{\alpha\beta} \bar{u}_{\underline{b}}{}^{\bar{z}} \, , \end{aligned} \tag{3.14}$$

as well as

$$V_q \sigma^{\underline{a}} V_p = \gamma^b_{qp} u_b{}^{\underline{a}} - \frac{1}{2}(\delta + i\gamma^9)_{qp} \bar{u}^{\underline{a}\bar{z}} - \frac{1}{2}(\delta - i\gamma^9)_{qp} u^{\underline{a}\, z} \quad , \tag{3.15}$$

where γ^a_{qp} are $d = 8$ gamma matrices and $\sigma^{\underline{b}}_{\alpha\beta}$ are the real 16×16 sigma-matrices of the SO(1,9) (see appendix A; more details on the moving frame variables can be found in appendix C).

Using this spinorial moving frame matrix one can convert the pull-backs of the fermionic target space supervielbein forms into the one-forms with the SO(1,7) spinorial index,

$$\hat{E}^{q1} := \hat{E}^{\alpha 1} V_\alpha{}^q \, , \qquad (\hat{E}^{q1})^* = \gamma^0_{qp} \hat{E}^{p1} \, , \tag{3.16}$$

$$\hat{E}^{q2} := \hat{E}^{\alpha 2} V_\alpha{}^q \, , \qquad (\hat{E}^{q2})^* = \gamma^0_{qp} \hat{E}^{p2} \, . \tag{3.17}$$

The worldsheet fermionic supervielbein form e^q can, in principle, be identified with any of these two one-forms, or with a linear combination of them. For D7-brane a convenient conventional constraint has the form

$$\hat{E}^{q1} := e^q \, . \tag{3.18}$$

Then the general form of the second fermionic supervielbein is[12]

$$\hat{E}^{q2} := e^p h_p{}^q + e^a \chi_a{}^q \; = \; \hat{E}^{p1} h_p{}^q + \hat{E}^a \chi_a{}^q \quad . \tag{3.19}$$

3.3 Consequences of the superembedding equation

The consistency conditions for the superembedding equation (3.11) read $d\hat{E}^z = 0$. To write them in the covariant form, it is convenient to introduce, besides the superspace spin connection $w^{\underline{ab}} := dZ^{\underline{M}} w_{\underline{M}}^{\underline{ab}}(Z)$, also the worldvolume connections for the SO(1,7) and SO(2) = U(1) gauge symmetries, $\omega^{ab} := d\zeta^{\mathcal{M}} \omega_{\mathcal{M}}{}^{ab}(\zeta)$ and $A := \zeta^{\mathcal{M}} A_{\mathcal{M}}(\zeta)$. These can be constructed with the use of the spinor moving frame variables in such a way that the covariant derivatives of the orthogonal and tangential moving frame vectors read (see [8, 16, 17] for more discussion in other examples)

$$Du_{\underline{a}}^{\;z} = u_{\underline{a}b} \Omega^{bz} \; , \qquad D\bar{u}_{\underline{a}}^{\;\bar{z}} = u_{\underline{a}b} \bar{\Omega}^{b\bar{z}} \; , \qquad Du_{\underline{a}}^{\;b} = \frac{1}{2} u_{\underline{a}}^{\;z} \bar{\Omega}^{b\bar{z}} + \frac{1}{2} \bar{u}_{\underline{a}}^{\;\bar{z}} \Omega^{bz} \; . \tag{3.20}$$

[12] To describe an SD7-brane, which is related to the standard D7-brane by a certain SL(2) transformation, it is convenient, following [19], to use the counterpart of (3.18) imposed on the SL(2) transformed doublet (E^1, E^2). Namely, in the notation of [19],

$$\hat{E}^{qi} := e^q v^i + (e^p h_p{}^q + e^a \chi_a{}^q) \tilde{v}^i \; , \qquad \tilde{v}^i = \epsilon^{ij} v_j \; , \qquad i,j = 1,2 \; ,$$

where v^i is the vector constructed from the axion and dilaton, and $\tilde{v}^i = \epsilon^{ij} v_j$ is its complementary vector. We however, will not use this constraint in the present paper; for our purposes here the mere fact of the existence of the SL(2) covariant formalism of [19] will be sufficient.

The 1-forms $\Omega^{bz}, \bar{\Omega}^{b\bar{z}}$ generalize (to the case of curved target superspaces) the Cartan forms corresponding to the $\mathrm{SO}(1,9)/[\mathrm{SO}(1,7)\times\mathrm{SO}(2)]$ coset.

3.3.1 Generalized Cartan forms, Peterson-Codazzi, Gauss and Ricci equations

Eqs. (3.20) provide us with conventional constraints, $\bar{u}^{\underline{a}\bar{z}}Du_{\underline{a}}^{\ z}=0$ and $u^{\underline{c}a}Du_{\underline{a}}^{\ b}=0$, expressing the worldvolume $\mathrm{U}(1)=\mathrm{SO}(2)$ and $\mathrm{SO}(1,7)$ connections, A and ω^{ab}, in terms of the pull-back $\hat{w}^{\underline{ab}}:=w^{\underline{ab}}(\hat{Z})$ of the target space spin connection $w^{\underline{ab}}(Z):=dZ^{\underline{M}}w_{\underline{M}}^{\underline{ab}}(Z)$ and moving frame variables u (entering through the true Cartan forms $u^T du$).

Similarly to (3.20), the covariant derivative of the spinor moving frame matrix, which is the element of $\mathrm{Spin}(1,9)$ covering the $\mathrm{SO}(1,9)$ matrix U, eq. (3.14), is given in terms of the same generalizations of the Cartan forms by

$$DV_\alpha{}^q=\frac{1}{4}\,\Omega^{az}\;V_\alpha{}^p(\gamma_a(\delta+i\gamma^9))_{pq}+\frac{1}{4}\,\bar{\Omega}^{a\bar{z}}\;V_\alpha{}^p(\gamma_a(\delta-i\gamma^9))_{pq}\,. \tag{3.21}$$

This relation, which expresses the local isomorphism of the SO(1,9) and Spin(1,9) groups, can be derived by solving the equation obtained by covariant differentiation of (3.14) with the use of (3.20).

The selfconsistency conditions for eqs. (3.20) give the following curved space generalization of the Maurer-Cartan equations

$$D\Omega^{az}=(u\hat{R}u)^{az}:=R^{\underline{cb}}u_{\underline{c}}{}^a u_{\underline{b}}{}^z\,,\qquad D\bar{\Omega}^{a\bar{z}}=(uRu)^{a\bar{z}}\,, \tag{3.22}$$

as well as the following expressions for the $\mathrm{U}(1)$ and $\mathrm{SO}(1,7)$ curvatures

$$\begin{aligned} r^{ab}:=d\omega^{ab}-\omega^{ac}\wedge\omega_c{}^b &= (uRu)^{ab}+\Omega^{[a|z}\wedge\bar{\Omega}^{|b]\bar{z}}\,,\\ dA &= \frac{i}{2}(uRu)^{\bar{z}z}+\frac{1}{2}\Omega^{az}\wedge\bar{\Omega}_a{}^{\bar{z}}\,, \end{aligned} \tag{3.23}$$

Eqs. (3.22), (3.23) involve the pull-back $\hat{R}^{\underline{ab}}:=R^{\underline{ab}}(\hat{Z})$ of the curvature $R^{\underline{ab}}:=(dw-w\wedge w)^{\underline{ab}}$ of the targets superspace spin connections $w^{\underline{ab}}$. They are the supersymmetric — and also curved superspace — generalizations of the Peterson-Codazzi, Ricci and Gauss equations [8] written for the case of codimension 2 supermebedding.

3.3.2 Consequences of the superembedding equation

Now we are ready to study the consistency conditions for the superembedding equation (3.11). Their covariant form is given by $D\hat{E}^z=0$ and the complex conjugate equation $D\hat{E}^{\bar{z}}=0$. Using eq. (3.20) and the bosonic torsion constraints (1.2),

$$T^{\underline{a}}:=DE^{\underline{a}}=-i(E^{\alpha1}\wedge E^{\beta1}-E^{\alpha2}\wedge E^{\beta2})\sigma^{\underline{a}}_{\alpha\beta}=:-iE^1\wedge\sigma^{\underline{a}}E^1-iE^2\wedge\sigma^{\underline{a}}E^2\,, \tag{3.24}$$

one finds that the coefficient $h_p{}^q$ in the decomposition of the pull-back of the second fermionic supervielbein form $E^{\alpha2}$ obeys

$$h(\delta\pm i\gamma^9)h^T=-(\delta\pm i\gamma^9) \tag{3.25}$$

and also that the curved space generalizations of the $\frac{SO(1,9)}{SO(1,7)\times SO(2)}$ Cartan forms (3.20) have the form of

$$\Omega^{az} = -2ie^q[h(\delta + i\gamma^9)\chi^a]_q + e_b(K^{ba\, z} - i\chi^b(\delta + i\gamma^9)\chi^a) \, , \qquad K^{ab\, z} = K^{(ab)\, z} \quad (3.26)$$

and of its complex conjugate. Notice that, substituting (3.26) into the Peterson-Codazzi equation (3.22), one finds that its lower dimensional component reads

$$D_{(p}(h(\delta + i\gamma^9)\chi_a)_{q)} + \frac{1}{2}(\gamma^b + h\gamma^b h^T)_{pq}(K_{ba}{}^z - i\chi_b(\delta + i\gamma^9)\chi_a) = \frac{i}{4}(\hat{R}^{az})_{pq} \, , \quad (3.27)$$

where $(\hat{R}^{az})_{pq} = (\hat{R}^{\underline{ab}})_{pq} u_{\underline{a}}{}^a u_{\underline{b}}{}^z$ and $\hat{R}^{\underline{ab}}_{pq}$ appears as a lowest dimensional coefficient in the decomposition of the pull-back of Riemann curvature two form on the basis of worldvolume supervielbein, $\hat{R}^{\underline{ab}} := \frac{1}{2}\hat{E}^{\underline{D}}\wedge\hat{E}^{\underline{C}}R_{\underline{CD}}{}^{\underline{ab}}(\hat{Z}) = \frac{1}{2}e^q\wedge e^p(\hat{R}^{\underline{ab}})_{pq} + \propto e^a$. It is expressed through pull-backs of the tangent superspace fluxes to the worldvolume superspace. We will not need the explicit form of $(\hat{R}^{az})_{pq}$ in this paper.

Eq. (3.27) relates the derivative of the fermionic superfield $\chi_a{}^q = \hat{E}_a^{\underline{\alpha}2} V_{\underline{\alpha}}{}^q$ with the bosonic symmetric tensor superfield

$$K_{ab}{}^z = K_{ba}{}^z := -D_{(a}\hat{E}_{b)}{}^{\underline{a}} u_{\underline{a}}{}^z \, , \quad (3.28)$$

which enters eq. (3.26). Writing this in terms of supervielbein pull-back, we have used the conventional constraint (3.8) in its equivalent form of $u_a{}^{\underline{b}} = E_a{}^{\underline{b}}$.

In the purely bosonic case, when fermions are equal to zero, $K_{ab}{}^z$ is called the *second fundamental form* of the bosonic worldvolume W^8 embedded into the $D = 10$ spacetime. Its trace $h^z := K_a{}^{az}$ is called *mean curvature*. The vanishing of the main curvature of a bosonic surface embedded in a flat space of higher dimension implies that this surface is *minimal*. This also expresses, in terms of *extrinsic geometry*, the equation of motion which follow form the Nambu-Goto action. (Which explains the terminology: minimal surface is obtained by minimizing the area (volume) of the surface (hypersurface)).

In the case of flat superspace and vanishing worldvolume gauge fields, the equations of motion of a superbrane also imply the vanishing of the *mean curvature*, $h^z := K_a{}^{az} = 0$. Clearly, in the case of generic curved superspace and of branes with additional worldvolume gauge fields, bosonic equations of motion for the scalar Goldstone (super)fields of the branes should be given by a nonlinear generalization of this, including the pull-backs of target superspace fluxes and also the worldvolume gauge field contributions. Furthermore, eq. (3.27) shows that such a bosonic equation can be obtained as higher component of the superfield fermionic equation, which in the week field limit (or for the simplest case of membrane in flat tangent superspace) has the form of Dirac equation $\gamma^a_{qp}\chi_a{}^p = 0$.[13] So, as is usual in supersymmetric theories, it is sufficient to find the superfield fermionic equation and then the bosonic scalar equations will appear in its decomposition in the Grassmann coordinates.

[13] Remember that $\chi_a{}^p = \nabla_a \hat{Z}^M E_M^{\underline{\alpha}2} V_{\underline{\alpha}}{}^p = \nabla_a \theta^{p\,2} + \ldots = \nabla_a W^q \gamma^9_{qp} + \ldots$; see section 3.1.2. Furthermore, ignoring the products of fields and the flux contributions, one finds that the linearized and flat superspace version of eq. (3.27) implies $K_a{}^{az} = -\frac{i}{16}D_q(\gamma^a\chi_a)_q$.

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However, neither such a fermionic equation nor bosonic equations of motion appear as a consequences of superembedding equation (3.11) for the IIB super-7-brane.

Thus to describe the type IIB 7-brane dynamics we have to search for additional constraints which would lead to the equations of motion.[14]

3.3.3 Selfconsistency of the equations for fermionic supervielbein forms

One should also study selfconsistency conditions for the fermionic equations (3.19), (3.18). Although (3.18) is a conventional constraint and (3.19) just manifest the general decomposition of the second fermionic supervielbein form on the basis of the worldvolume superspace one forms, such integrability conditions provide us with the properties of ${h_p}^q$ and χ_a^q superfields which they possesses by their definition in eqs. (3.19), (3.18). To clarify this point, let us notice that in the linearized approximation and in flat superspace ${h_p}^q = D^0_p\,\theta^{2q}$ and ${\chi_a}^q = \partial_a\theta^{2q}$ and, hence, $D^0_{(p}\,{h_{p')}}^q = 2i\gamma^a_{pp'}\chi^q_a$ and ${\partial_{[a}\chi_{b]}}^q = 0$. The selfconsistency conditions for (3.19), (3.18) carry the nonlinear counterpart of these two equations valid in an arbitrary superspace supergravity background.

In particular, the lowest, 1/2 dimensional component of the integrability condition $D(E^{2q} - e^p{h_p}^q - e^a\chi^q_a) = 0$ for eq. (3.19) produces the expression for ${D_{(p}h_{p')}}^q$ (see eq. (D.9) in appendix D), the trace part of which reads

$$D_p{h_p}^q = -14\left({(hV)_q}^\alpha\widehat{D_{\alpha 1}e^{-\Phi}} + {V_q}^\alpha\widehat{D_{\alpha 2}e^{-\Phi}}\right) \ , \tag{3.29}$$

where $\widehat{D_{\alpha 1,2}e^{-\Phi}}$ are the pull-backs of $D_{\alpha 1,2}e^{-\Phi}$ superfield to $W^{(8|16)}$. Below we will omit hat symbols from the pull-backs in the places where this cannot produce a confusion.

Eq. (3.29) is not dynamical when considered together with superembedding equation only. However, as we will discuss below, after imposing the D7-brane gauge field constraint an additional algebraic equation for h appears (see eq. (3.34) below). Considered together with this, eq. (3.29) becomes dynamical and, moreover, collecting all the set of the dynamical equations of motion.

3.4 Superspace constraints for D7-brane worldvolume gauge field

In the case of M-branes, D=10 fundamental string and Dp-branes with $p \leq 5$ the superembedding equation contains equations of motion among their consequences [8, 10, 15]. Hence, the description of *the set of all possible p-branes* by this equation is complete. Then what happens if several type of p-branes are possible? In [17] this question was addressed for the strings (1-branes) in type IIB superspace; it was shown there that the superembedding equation provides a universal description of fundamental string (sometimes called F1-brane) and Dirichlet 1-brane (D1-brane).[15] Similarly, the superembedding equation can provide the universal description of all possible type IIB 5-branes, so that the search for

[14]In the light of the off-shell nature of the superembedding equation for type IIB 7-branes, one could also search for a superfield action of STV type (see [22, 25] and [16] for review and further references) producing these equations of motion together with the superembedding equations. We do not try to elaborate this direction in the present paper.

[15]The generic solution of the superembedding equation for $p = 1$ describes the case of super-D1-brane, and the worldvolume gauge filed strength enter the solution as a parameter. In the simplest case of flat

possible existence of new 5-branes (if any) can be performed by searching for possible existence of new solutions of the superembedding equation, different from the ones describing D5-brane and its SL(2) images (including NS5; these are called (p,q) five branes, although we could also propose the name of SD5-brane).

This is not the case when $p = 7$. The superembedding equation for this case is not dynamical and, to describe the dynamics of D7-brane, the constraints on the worldvolume gauge field strength (2.14) should be imposed [15]. This constraint reads

$$\mathcal{F}_2 = dA - \hat{B}_2 = \frac{1}{2} e^b \wedge e^a F_{ab} \ , \tag{3.30}$$

where e^a is the worldvolume superspace bosonic vielbein induced by superembedding, eq. (3.8), and $\hat{B}_2$ is a counterpart of the WZ term of the fundamental string, but on the 7-brane worldvolume superspace. In general curved type IIB background this is given by the pull-back of the NS-NS two-form gauge field B_2, the field strength of which, $H_3 = dB_2$, is restricted by the following superspace constraints

$$H_3 := dB_2 = -iE^{\underline{a}} \wedge (E^1 \wedge \sigma_{\underline{a}} E^1 - E^2 \wedge \sigma_{\underline{a}} E^2) + \frac{1}{3!} E^{\underline{c}_3} \wedge E^{\underline{c}_2} \wedge E^{\underline{c}_1} H_{\underline{c}_1 \underline{c}_2 \underline{c}_3} \ . \tag{3.31}$$

In the case of flat type IIB superspace, the last, bosonic 3-form contribution vanishes, and eq. (3.31) is equivalent to the first component of the doublet equation (2.5), as it is indicated by (2.17).

Now, studying the Bianchi identities (2.14) for the worldvolume field strength one finds, besides the purely bosonic identity $3D_{[a}F_{bc]} = -H_{abc} - 6i(\eta + F)_{d[a}\, \chi_b \gamma^d \chi_{c]}$ and

$$D_q F_{bc} = -4i(\eta - F)_{a[b} (h\gamma^a \chi_{c]})_q \ , \tag{3.32}$$

that the spin-tensor $h_p{}^q$ in (3.19) obeys the algebraic equation

$$h\gamma^a h^T (\eta - F)_{ab} = \gamma^a (\eta + F)_{ab} \ . \tag{3.33}$$

For $F_{ab} = 0$ the system of this equation and eq. (3.25) is solved by $\overset{0}{h}_p{}^q = (\gamma^9)_{pq}$. In the generic case of nonvanishing F_{ab}, eq. (3.33) can be written as

$$h\gamma^a h^T = \gamma^b k_b{}^a \ , \qquad k_b{}^a := (\eta + F)_{bc} (\eta - F)^{-1\, ca} \ \in \ \mathrm{SO}(1,7) \ . \tag{3.34}$$

This includes the pseudo-orthogonal 8×8 matrix $k_a{}^b$ ($k\eta k^T = \eta$) the Cayley image of the antisymmetric tensor F_{ab}. Eq. (3.33) implies that h is an element of the Spin$(1,7)$ group. Eqs. (3.33) and (3.25) are solved by

$$\begin{aligned} h_p{}^q = \frac{1}{\sqrt{|\eta + F|}} \Big[\gamma^9 &+ \frac{1}{2} F_{ab} \gamma^{ab} \gamma^9 - \frac{1}{8 \cdot 4!} (\varepsilon FF)^{a_1 a_2 a_3 a_4} \gamma_{a_1 a_2 a_3 a_4} + \\ &+ \frac{1}{4 \cdot 4!} (\varepsilon FFF)^{ab} \gamma_{ab} - \frac{1}{16 \cdot 4!} (\varepsilon FFFF)\, \delta \ \Big]_{pq} \ . \end{aligned} \tag{3.35}$$

superspace this is just a constant parameter, the appearance of which had been observed already in [8]. From the side of this generic D1-brane case, the solution describing the fundamental string appears in the limit when the D1-brane field strength reaches its maximal possible value [17]. Notice that another universal description of string and D1-brane is provided by SL(2) covariant superstring action of [45].

More discussion on the Lorentz group valued spin tensor h for different Dp-branes can be found in [14, 47]. Special properties of the $p=7$ case are related to that, due to (3.25),

$$h^T h = -I = hh^T \, , \qquad h\gamma^9 h^T = -\gamma^9 \, , \tag{3.36}$$

so that the matrix inverse to h is given by $-h^T$. This implies

$$h\gamma^{a_1\ldots a_n} h^T = (-)^{n+1} \gamma^{b_1\ldots b_n} k_{b_1}{}^{a_1} \ldots k_{b_1}{}^{a_n} \, , \quad h\gamma^{a_1\ldots a_n}\gamma^9 h^T = (-)^n \gamma^{b_1\ldots b_n}\gamma^9 \, k_{b_1}{}^{a_1} \ldots k_{b_1}{}^{a_n} \, . \tag{3.37}$$

3.5 D7-brane equations of motion from gauge field constraints plus superembedding equations

Eq. (3.34) implies that $h^{-1}dh$ takes values in $spin(1,7)$ Lie algebra, $k^{-1}dk$ takes its values in so$(1,7)$ and

$$h^{-1}Dh = \frac{1}{4}(k^{-1}Dk)^{ab}\gamma_{ab} \, . \tag{3.38}$$

Then, as far as, by construction, $(k^{-1}Dk)^{ab} = 2(\eta+F)^{-1}DF\,(\eta-F)^{-1}$, (3.38) implies

$$h^{-1}Dh = \; \frac{1}{2}\, DF_{ab}(\eta-F)^{-1ac}(\eta-F)^{-1bd}\gamma_{cd} \, , \tag{3.39}$$

$$Dh\, h^{-1} = -\frac{1}{2} DF_{ab}(\eta+F)^{-1ac}(\eta+F)^{-1bd}\gamma_{cd} \, . \tag{3.40}$$

Eq. (3.40) is obtained from (3.39) using eq. (3.37) with $n=2$.

Now, substituting (3.32) into the fermionic component of eq. (3.40), one finds $(D_q h\, h^{-1})_{p'q'} = -2i(h\gamma^a\chi_c)_q(\eta-F)^{-1cb}\,(h\gamma_{ab}h^T)_{p'q'}$. Taking into account that $h^T=-h^{-1}$, one can present this equation in the form of $(D_q h)_{p'p} = 2i(h\gamma^a\chi_c)_q(\eta-F)^{-1cb}\,(h\gamma_{ab})_{p'p}$; the trace of this gives

$$D_q h_q{}^p \; = -14i(\eta-F)^{-1ab}(\gamma_b\chi_a)_p \, . \tag{3.41}$$

However, $D_q h_q{}^p$ is expressed in terms of pull-backs of background superfields by the consequence (3.29) of eqs. (3.19), (3.18). Thus we arrive at

$$(\eta-F)^{-1ab}\gamma_{bqp}\chi_a{}^p = -i\left((hV)_q{}^\alpha D_{\alpha 1}e^{-\Phi} + V_q{}^\alpha D_{\alpha 2}e^{-\Phi}\right) \, , \qquad \chi^p_a := \hat{E}^{\alpha 2}_a V_\alpha{}^p \tag{3.42}$$

which is the (superfield generalization of the) fermionic equation of motion for D7-brane.[16]

In the *flat D=10 type IIB superspace* $D_{\alpha 1}e^{-\Phi} = 0 = D_{\alpha 2}e^{-\Phi}$, so that the *r.h.s*'s of both eq. (3.42) and eq. (3.29) vanish. Hence the latter equation simplifies to $D_q h_q{}^p = 0$ and eq. (3.42) can be written as

$$\gamma_{aqp}(\eta+F)^{-1ab}D_b\Theta^{2\,p} = 0 \tag{3.43}$$

(notice that $(\eta-F)^{-1ba} = (\eta+F)^{-1ab}$) . This differs from the standard Dirac equation by the contribution of the worldvolume flux F_{ab}: decomposing (3.43) in power series on F one easily finds $\gamma^a{}_{qp}D_a\Theta^{2\,p} = \gamma_{aqp}F^{ab}D_b\Theta^{2\,p} + \mathcal{O}(FF)$.

[16] See [12, 14, 47] for the Dp-brane equations and earlier [10] for the M5-brane case.

As usually, the bosonic equations of motion can be obtained by acting on the fermionic equations by covariant spinor derivative. On this way, one should use also the algebraic consequences of the superembedding equations, including eq. (3.27). We will not need in the explicit form of the D7-brane bosonic equations in our discussion below. See [47] for detailed study of the (component form of the) type IIB Dp-brane equations.

4 Searching for a superembedding description of Q7-brane

The discussion in our section 2 supports the conclusion of [2] that the Q7-branes (if exist as dynamical objects) should carry two gauge fields on their worldvolume.

As far as a complete supersymmetric description is concerned, the problem with counting bosonic and fermionic degrees immediately arise. In the case of D7-brane (and SD7-branes of [19]) the number of dynamical bosonic and fermionic degrees of freedom coincide $(2 + 6 = 16/2)$ and the gauge field degrees of freedom enter the balance (as '6'). Now, adding a new gauge field, one should either add for him additional fermionic degrees of freedom or to assume its nondynamical/dependent nature. As a mechanism for the latter, the authors of [2] suggested a possibility of some generalization of duality equations. However, no mechanism to generate such generalized duality equations is know. Let us note in this respect that it is natural to assume that, if existed, such description with self-duality equations should be also reproducible in the frame of superembedding approach, like, for instance, the self-duality of the five-form field strength follow from the superspace constraints of type IIB supergravity [40] or like the nonlinear generalization of the six dimensional self-duality equation follows from the superembedding equation of M5-brane [10] (see [11, 48] for details).

The above statement actually is based on more general conjecture: *if existed, the Q7-brane should also allow for a superembedding description.* The reason for this is that it was the case for all previously known branes; more generally, to our best knowledge, there is no one known example of a supersymmetric system (neither field theoretical nor brane-type) which do not allow for an *on-shell* superfield description.

Furthermore, it is also natural to assume that the Q7-brane worldvolume superspace obeys the superembedding equation, eq. (3.4) or equivalently (3.11). Again, the reason is that this is the case for the (complete) superfield description of all the presently known 1/2 BPS superbranes (which means super-p-branes the ground state of which preserves one half of the target space supersymmetry), and the supergravity solutions describing Q7-branes are 1/2 BPS [1, 3].[17]

Then, the worldvolume superspace of the Q7-brane, if existed, would carry two 'linearly independent' worldvolume gauge potential super-1-forms, i.e. two super-1-forms with generalized field strengths given by $q_1^R \mathcal{F}_R$ and $q_2^R \mathcal{F}_R$ with linearly independent doublets of

[17]See [49] for the relation between projectors of the κ-symmetry of brane actions and supersymmetry preserved by corresponding bosonic solutions of supergravity equations, [50] for the supersymmetry preserved by bosonic brane actions (bosonic limit of superbrane action) and [51] for the complete but gauge fixing Lagrangian description of the supergravity-superbrane interacting system, which explains the above mentioned relation between the supersymmetry and the κ-symmetry.

charges, q_1^R and q_2^R, and $\mathcal{F}_R$ defined in (2.16). Notice that, as we discussed in section 2.3, such a straightforward lifting to worldvolume superspace is what is necessary for having a possibility to construct the standard-type Q7-brane action by superembedding approach based method of [30].

To simplify our study, let us consider a particular situation when one of two worldvolume gauge fields of the hypothetical Q7-brane is the familiar gauge field of D-brane, $A = d\zeta^M A_M(\zeta)$, related to the pull-back of NS-NS form, (3.30),

$$\mathcal{F}_2 = dA - \hat{B}_2 \ , \tag{4.1}$$

while the other, which we call $A^{(q,q')} = d\zeta^M A_M^{(q,q')}(\zeta)$, has its field strength defined by

$$G_2 := dA^{(q,q')} - q\hat{B}_2 - q'\hat{C}_2 \tag{4.2}$$

with some constants q and $q' \neq 0$. The existence of the SL(2) covariant formalism [19] guaranties that the general situation can be reproduced by certain SL(2) transformation of the above choice (see also comment in footnote 12).

The field strength G_2 defined by eq. (4.2) is invariant, besides the abelian worldvolume gauge transformations, under the NS-NS and RR target superspace gauge transformations,

$$\delta B_2 = d\alpha_1 \ , \qquad \delta C_2 = d\alpha_1' \ , \qquad \delta A^{(q,q')} = q\hat{\alpha}_1 + q'\hat{\alpha}_1' + d\alpha_0 \ . \tag{4.3}$$

Similarly, the invariance of the field strength $\mathcal{F}_2$ is guaranteed by that $\delta A = \hat{\alpha}_1$; this, in contrast to generic $A^{(q,q')}$ with $q' \neq 0$, is inert under the RR gauge transformations.

Now the question arise: what are the constraints which should be imposed on these two field strengths?

4.1 Candidate constraints for two worldvolume gauge potentials on the worldvolume superspace

A strong suggestion comes form the linearized analysis. Indeed, if there exists a worldvolume model of Q7-branes with two dynamical gauge fields, this should allow, in particular, for zero values of the fields, and, hence, for a weak field approximation. Thus it is reasonable to search first for the linearized description of the two independent and different gauge fields on a 7-brane worldvolume.

Then one can check that the following constraints

$$\mathcal{F}_2 = \frac{1}{2} e^b \wedge e^a F_{ab} \tag{4.4}$$

$$G_2 = \frac{1}{2} e^b \wedge e^a G_{ab} + e^b \wedge e^q \gamma_{b\,qp} \mathcal{W}^p + \frac{1}{2} e^p \wedge e^q ((\delta + i\gamma^9)_{pq} \Upsilon + (\delta - i\gamma^9)_{pq} \bar{\Upsilon}) \ , \tag{4.5}$$

have a correct weak field limit. Namely, they, together with the linearized superembedding equations result in the Dirac equation for the fermionic fields, Klein-Gordon equations for scalars and Maxwell equations for the vector fields of two d=8 vector multiplets,[18]

$$
\begin{array}{ccc|cc}
\multicolumn{2}{c}{\text{Superembedding multiplet}} & & \multicolumn{2}{c}{\text{Additional vector multiplet}} \\
\partial_{[c}F_{ab]}=0\,, & \partial^b F_{ab}=0\,, & & \partial_{[c}G_{ab]}=0\,. & \partial^b G_{ab}=0\,, \\
 & \not\partial_{pq}W^q=0\,, & & \not\partial_{pq}\mathcal{W}^q=0\,, & \\
\Box\tilde{X}^z=0\,, & \Box\tilde{\bar{X}}^{\bar{z}}=0\,, & & \Box\Upsilon=0\,, & \Box\bar{\Upsilon}=0\,.
\end{array}
\qquad \begin{array}{r} \\ (4.6) \\ (4.7) \\ (4.8) \end{array}
$$

Here we separated the field equations in two blocks coorresponding to two different vector multiplets. The first of these two d=8 vector multiplets is formed by leading components of the Goldstone superfields of eq. (3.7) and of the antisymmetric tensor superfield F_{ab} in (4.1). We call this the *superembedding multiplet* and its equations of motion are actually the linearized equations of motion of the super-D7-brane. The second $d=8$ linearized vector multiplet is formed by leading components of the G_{ab}, $\Upsilon=(\bar{\Upsilon})^*$ and $\mathcal{W}^q$ superfields in (4.5). In the linearized approximation this constraints is considered on the flat $d=8$ superspace because the contributions to the worldvolume geometry from the superembedding multiplet are neglected as being of second order in fields.

It is a place to stress that, in the 'rigid' Q7-brane picture it is hard to find a place for the 2 bosonic and 16 fermionic fields Υ and $\mathcal{W}^q$ (we use the same notation for the superfields and their leading components when this cannot produce a confusion). On the other hand, such fields look quite natural if we think about Q7-brane as about bound state of two SD7-branes (one of which is identified, for simplicity, with D7-brane). Then these additional fields Υ and $\mathcal{W}^q$ complete the additional bosonic field G_{ab} up to a vector supermultiplet which is identified as the superembedding supermultiplet of the second SD7-brane.

The problem of superpartners of the second gauge field on the hypothetical Q7-brane, have been noticed in [2]. To escape the treatment of Q7 as a system of two SD7-branes it was proposed there that the two gauge field strengths are related by a kind of nonlinear d=8 generalization of the selfduality condition, like, schematically, $*G\propto F\wedge F\wedge F$ or $G\wedge G=*(F\wedge F)$,[19] so that the second set of superpartners is not needed.

No dynamical mechanism for generating such a nonlinear generalized duality equation was proposed in [2]. But, if existed, such equation should follow from superembedding approach, i.e. appear as a requirement of selfconsistency of the system including superembedding equation and the gauge field constraints *without* additonal superfields Υ and $\mathcal{W}^q$. If this were the case, then the addition of unwanted bosonic and fermionic (super)fields

[18] It is also worth noticing that the set of constraints (4.4), (4.5) have the SL(2) covariant generalization which, in notation of [19], reads

$$
\mathcal{F}^{(2)}_r=\frac{1}{2}e^b\wedge e^a\mathcal{F}_{ab\,r}+\tilde{V}_r\left(e^b\wedge e^q\gamma_{b\,qp}\mathcal{W}^p+\frac{1}{2}e^p\wedge e^q((\delta+i\gamma^9)_{pq}\Upsilon+(\delta-i\gamma^9)_{pq}\bar{\Upsilon})\right)\,,
$$

where $\tilde{V}_r$ is a 2-dimensional SO(2) vector constructed from axion and dilaton, namely $\tilde{V}_r=\epsilon^{rs}V_s$ with $V_r=\sqrt{2}v^i(\tau_r)_{ij}\tilde{v}^j$, and the SO(2) spinors v^i and $\tilde{v}^i=\epsilon^{ij}v_j$ constructed from the axion and dilaton; these are the ones used in an SL(2) covariant formulation of the fermionic conventional constraints (3.18) and (3.19), $\hat{E}^{qi}:=e^qv^i+(e^ph_p{}^q+e^a\chi_a{}^q)\tilde{v}^i$ [19], discussed in footnote 12.

[19] More precisely, the equations proposed in [2] were $\mathcal{G}^-\wedge\mathcal{G}^-=*\mathcal{G}^-\wedge\mathcal{G}^-$ with $\mathcal{G}^\pm=G\pm\mathcal{F}$, and $\mathcal{G}=*(\mathcal{G}\wedge*\mathcal{G}\wedge\mathcal{G})$, but the explicit forms is not essential for our present study.

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Υ and $\mathcal{W}^q$ cannot spoil the results: these fields would either be set to zero by the above mentioned selfconsistency conditions, or allow for being set to zero at the final result.[20] The same analysis also shows that the linearized G_{ab} vanishes if $\Upsilon = 0$ and $\mathcal{W}^q = 0$; this excludes the possibilities of generating equations of the type of $G \wedge G = *(F \wedge F)$, but still allows to conjecture the appearance of e.g. $*G =\propto F \wedge F \wedge F$ equation, as far as its linearized limit would be just $G_{ab} = 0$. This conjecture on possible appearance of such a duality conditions can now be checked.

To this end we shall study the selfconsistency conditions for the superembedding equation (3.11) and the constraints (4.4), (4.5), the form of which have been motivated by the consistency of the linearized approximation. Actually, the consistency of the superembedding equations (3.11) and the constraints (4.4) for the first worldvolume gauge field strength has been already checked in section 3. This consistency requires the D7-brane equations of motion to hold.

Thus *the search for a Q7-brane description in the frame of superembedding approach is reduced to checking of whether it is possible to have the second different gauge field super-1-form potential on the worldvolume superspace of the D7-brane.* Furthermore, it actually reduces to checking of whether the constraints (4.5) can be consistent on such a worldvolume superspace.

The result of such a check is negative. Taking into account the importance of this conclusion, we present below some technical details.

4.2 Solving the Bianchi identities for the second gauge field on the worldvolume superspace of D7-brane

Using the consequences $\gamma^a + h\gamma^a h^T = 2\gamma_b(\eta - F)^{-1ba}$, and $\gamma^a - h\gamma^a h^T = -2\gamma^b\, F_{bc}(\eta - F)^{-1ca}$ of eq. (3.34), one finds that the lowest dimensional (dim 3/2) contribution to the Bianchi identities

$$dG_2 + (q + q'\hat{C}_0)\hat{H}_3 + q'\hat{R}_3 = 0 \tag{4.9}$$

for the constraints (4.5), (4.2) reads

$$\begin{aligned} 2i\gamma_{b\,(q_1q_2}(\eta - F)^{-1ba}(\gamma_a\mathcal{W})_{q_3)} + \frac{1}{2}(\delta + i\gamma^9)_{(q_1q_2}D_{q_3)}\Upsilon + \frac{1}{2}(\delta - i\gamma^9)_{(q_1q_2}D_{q_3)}\bar{\Upsilon} = \\ = -\frac{1}{2}T_{(q_1q_2}{}^{p}\left((\delta + i\gamma^9)_{q_3)p}\Upsilon + (\delta - i\gamma^9)_{q_3)p}\bar{\Upsilon}\right)\ . \end{aligned} \tag{4.10}$$

The explicit form of the worldvolume fermionic torsion $T_{pq}{}^{q'}$ can be found in appendix D (see eq. (D.3) and also (C.25)). Decomposing eq. (4.10) on the irreducible parts, one finds (see appendix E for details)

$$D_q\Upsilon = -2i(\delta - i\gamma^9)_{qp}\left(\mathcal{W}^p - \Upsilon\Lambda^1_p\right)\ , \tag{4.11}$$

[20] One might hope that these would be auxiliary fields providing the off-shell extension of the first, superembedding supermultiplet, but such a possibility has been actually excluded by the linear approximation analysis which results in dynamical equations (4.6), (4.7), (4.8).

with

$$\Lambda^1_q := \frac{i}{2} V_q{}^{\alpha} (\widehat{D_{\alpha 1} e^{-\Phi}}) \, , \qquad \Lambda^2_q := \frac{i}{2} V_q{}^{\alpha} (\widehat{D_{\alpha 2} e^{-\Phi}}) \, , \tag{4.12}$$

and also

$$(F(\eta - F)^{-1})_b{}^c ((\delta + i\gamma^9)\gamma_c \mathcal{W})_p - 2((\delta + i\gamma^9)\gamma_b \Lambda_1)_p \, \Upsilon - ((\delta + i\gamma^9) h \chi_b)_p \, \bar{\Upsilon} = 0 \, . \tag{4.13}$$

In distinction to (4.11), eq. (4.13) is essentially nonlinear: it does not have a nontrivial linear approximation in the case of flat background superspace.

One also finds the equations complex conjugate to (4.11) and (4.13), so that it is possible to find, in particular, the expression for $(F(\eta - F)^{-1})_b{}^c (\gamma_c \mathcal{W})_q$. However, it is more convenient to use first the the dynamical equation for the superembedding fermion, which is just the D7-brane fermionic equation (3.42),

$$(\eta - F)^{-1ba} \gamma_a \chi_b = -2(\Lambda^2 + h\Lambda^1) \, , \tag{4.14}$$

and to write eq. (4.13) in the equivalent form

$$(F(\eta - F)^{-2})^{ab} (\delta - i\gamma^9)\gamma_a\gamma_b \mathcal{W} = 2(\delta - i\gamma^9) \left(\Upsilon \gamma_a (\delta - F)^{-1ab} \gamma_b \Lambda^1 + \bar{\Upsilon} h (\Lambda^2 + h\Lambda^1) \right) . \tag{4.15}$$

In the derivation of (4.15) one uses eq. (3.34) and (3.36). Notice that r.h.s. of this equation is proportional to the pull-backs of spinorial derivatives of the dilaton superfield (4.12), which can be called *fermionic fluxes*, and, hence, vanishes for the 7-brane in flat target superspace where (4.15) and its complex conjugate imply $(F(\eta - F)^{-2})^{ab} (\gamma_a \gamma_b \mathcal{W})_q = 0$. This equation stating vanishing of the product of fields from different supermultiplets already suggests a possibility of that these cannot coexist. But to see that this is indeed the case we have to go through further studying the consequences of eq. (4.15) and the dim 2 component of the Bianchi identities (4.9).

Acting by the fermionic derivative D_p on (4.15), one finds[21]

$$(F(\delta - F)^{-2})^{ab} (\gamma_a \gamma_b)_{pq'} D_q \mathcal{W}^{q'} + \mathcal{O}(\Upsilon) + \mathcal{O}(\mathcal{W}) = 0 \, . \tag{4.16}$$

For simplicity we do not write in eqs. (4.20) and (4.16) an explicit form of the terms proportional to scalar and spinor superfields, $\mathcal{O}(\Upsilon)$ and $\mathcal{O}(\mathcal{W})$. In this notation, the real part of eq. (4.15) reads

$$(F(\eta - F)^{-2})^{ab} (\gamma_a \gamma_b \mathcal{W})_q + \mathcal{O}(\Upsilon) = 0 \, . \tag{4.17}$$

Notice that the terms denoted here by $\mathcal{O}(\Upsilon)$ are also proportional to the fermionic fluxes (4.12) and vanish for the case of flat superspace. To stress this one might use an alternative form of writing eqs. (4.16) and (4.17),

$$(F(\eta - F)^{-2})^{ab} (\gamma_a \gamma_b \mathcal{W})_q + \mathcal{O}(\Lambda) = 0 \, , \tag{4.18}$$

$$(F(\delta - F)^{-2})^{ab} (\gamma_a \gamma_b)_{pq'} D_q \mathcal{W}^{q'} + \mathcal{O}(\mathcal{W}) + \mathcal{O}(\Lambda) + \mathcal{O}(D_q \Lambda) = 0 \, . \tag{4.19}$$

[21] In derivation of eq. (4.16) one have also take into account eqs. (3.32) and (3.39).

Using the worldvolume covariant derivative algebra (this is to say, torsion and curvature describing worldvolume geometry induced by the superembedding, see appendix D), one finds (from (4.11)) that

$$D_p \mathcal{W}^{q'} = i a_{ab} \gamma^{ab}_{pq'} + i\tilde{a}_{ab}(\gamma^{ab}\gamma^9)_{pq'} + \mathcal{O}(\Upsilon) + \mathcal{O}(\mathcal{W}) \tag{4.20}$$

(see eq. (E.9) of appendix E for a more complete form). Here a_{ab} and $\tilde{a}_{ab}$ are antisymmetric tensors which have to be determined from the further study of Bianchi identities; note that the terms denoted by $\mathcal{O}(\Upsilon) + \mathcal{O}(\mathcal{W})$ in (4.20) do not contain irreducible parts $\propto \gamma^{ab}_{pq'}$ and $\propto (\gamma^{ab}\gamma^9)_{pq'}$. Substituting (4.20), one finds that eq. (4.19) implies, in particular,

$$(F(\delta - F)^{-2})^{ab} a_{ab} + \mathcal{O}(\mathcal{W}) + \mathcal{O}(\Lambda) + \mathcal{O}(D_p\Lambda) = 0\,, \tag{4.21}$$
$$(F(\delta - F)^{-2})^{ab} \tilde{a}_{ab} + \mathcal{O}(\mathcal{W}) + \mathcal{O}(\Lambda) + \mathcal{O}(D_p\Lambda) = 0\,, \tag{4.22}$$

where we account separately for terms proportional to fermionic fluxes, $\mathcal{O}(\Lambda)$, and to the Grassmann derivatives of the fermionic fluxes, $\mathcal{O}(D_p\Lambda)$ as far as these latter collect contributions from the bosonic fluxes.

The next, dim 2 component of the Banchi identities (4.9) reads

$$\begin{aligned}&-4i\gamma_{c\,qp}\,(\eta - F)^{-1\,ca}(G_{ab} - \tilde{q}F_{ab}) + 4i\tilde{q}'(h\gamma_b)_{(qp)} + 2D_{(p}(\gamma_b\mathcal{W})_{q)} + \\ &+(\delta + i\gamma^9)_{qp} D_b\Upsilon + (\delta - i\gamma^9)_{qp} D_b\bar{\Upsilon} + \mathcal{O}(\mathcal{W}) + \mathcal{O}(\Upsilon) = 0\,,\end{aligned} \tag{4.23}$$

where we have introduced the notation of effective charges

$$\tilde{q} := q + q'\hat{C}_0\,, \qquad \tilde{q}' := q' e^{-\hat{\Phi}}\quad . \tag{4.24}$$

The second term in (4.23) can be specified with the use of the explicit solution (3.35) of eqs. (3.34), (3.25),

$$\begin{aligned}&(h\gamma_b)_{(pq)} = \\ &-\frac{1}{2\sqrt{|\eta+F|}}\left[(\gamma^{a_1a_2a_3}\gamma^9)_{pq}(\eta - F)_{b[a_1}F_{a_2a_3]} + \frac{1}{4!}\gamma^c_{pq}\left((\varepsilon FFF)_{bc} + \frac{1}{8}\eta_{bc}\varepsilon FFFF\right)\right],\end{aligned} \tag{4.25}$$

where $(\varepsilon FFF)_{ab} = \varepsilon_{abc_1\ldots c_6}F^{c_1c_2}F^{c_3c_4}F^{c_5c_6}$, etc. Using eqs. (4.25) and (4.20), one finds that the $\propto (\gamma^{c_1c_2c_3}\gamma^9)_{\,pq}$ irreducible part of eq. (4.23) reads, modulo terms proportional to scalar and spinor superfields,

$$\eta_{b[c_1}\tilde{a}_{c_2c_3]} = -\frac{\tilde{q}'}{\sqrt{|\eta+F|}}(\eta - F)_{b[c_1}F_{c_2c_3]}\,. \tag{4.26}$$

To be precise, this equation is valid in its literal form in the case of flat superspace, and modulo fermionic bilinear contributions; all but one terms in its actual r.h.s. are proportional either to the background superspace fluxes or to the pull-back of the target space fermionic superfields (4.12), and the only exception is $-\frac{1}{48}(\mathcal{W}\gamma_a\gamma_{c_1c_2c_3}\gamma^9 h\gamma^a\chi_b)$ (see eq. (E.10) in appendix E). Of course, this r.h.s. also vanishes when we set to zero the additional bosonic and fermionic superfields, $\Upsilon = 0 = \mathcal{W}^q$ (which are also not wanted from the point of view of the Q7-brane picture).

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An immediate consequence of eq. (4.26) is that

$$F_{[bc_1}F_{c_2c_3]} = 0\ . \tag{4.27}$$

Passing to a special Lorentz frame where F_{ab} has Darboux's standard form, $F_{ab} = i\sigma^2 \otimes \mathrm{diag}(f_1, f_2, f_3, f_4)$, one easily finds that, for any solution of eq. (4.26), only one of four 'eigenvalues' f_1, f_2, f_3, f_4 might be nonzero. In other words, the general solution of $F_{[bc_1}F_{c_2c_3]} = 0$ is given by $F_{ab} = 2k_{[a}l_{b]}\, f$ with $l^2 = l_al^a = -1$, $k_ak^a = \pm 1$ and an arbitrary function f. Then $F_{b[c_1}F_{c_2c_3]} = 0$ and, hence, $\tilde{a}_{ab} = -\frac{\tilde{q}'}{\sqrt{|\eta+F|}}F_{ab}$. To resume,

$$F_{ab} = 2k_{[a}l_{b]}\, f\ , \qquad l^2 = l_al^a = -1\ , \qquad k^2 = k_ak^a = \pm 1\ , \tag{4.28}$$

$$\tilde{a}_{ab} = -\frac{\tilde{q}'}{\sqrt{|\det(\eta + F)|}}\, F_{ab} = -\frac{\tilde{q}'\, 2k_{[a}l_{b]}f}{\sqrt{|1-f^2|}}\ . \tag{4.29}$$

Now, using the consequence (4.22) of eq. (4.19), which on the level of accuracy of our present discussion reads $(F(\delta - F)^{-2})^{ab}\tilde{a}_{ab} = 0$, one concludes that, for $\tilde{q}' \neq 0$,

$$f^2 = 0\ . \tag{4.30}$$

Thus the system of eq. (4.26) and (4.19) has only trivial solutions, so that $\tilde{a}_{ab} = 0$ and

$$\tilde{q}'F_{ab} = 0\ . \tag{4.31}$$

Of course, one might notice that our discussion at this point is rough enough as far as, in particular, in our superspace context, eq. (4.30) implies just the nilpotence of f rather than $f = 0$, so that eq. (4.31) should be rather replaced by the statement of vanishing of the 'pure bosonic body' (not nilpontent part) of F_{ab}. Next, even ignoring this simplest nilpotent contribution, one might note that, in the case of nonvanishing fermionic (super)fields and fluxes we should rather have $\tilde{q}'F_{ab} = \mathcal{O}(\Upsilon) + \mathcal{O}(\mathcal{W})$. But such a r.h.s. (either from the additional scalar and spinor fields Υ and $\mathcal{W}$, which do not have good treatment in discussions of hypothetical Q7-branes, or from the nilpotent f in (4.28)) does not prevent from the conclusion that the selfconsistency does not allow for a second (actually first) dynamical gauge field and that for $q' \neq 0$ *our constrains rather describe a fixed field configuration than a dynamical system with two gauge field potentials.*

4.3 Conclusion

Thus we have shown that, imposing the type IIB 7-brane superembedding equation together with two different sets of gauge field constraints, requires, instead of dynamical equations, that one of the field strengths vanish, $F_{ab} = 0$ (or expressed through the additional scalar and fermionic fields introduced artificially to complete the other field strength till a supermultiplet). This shows that there is no place for Q7-brane in the standard setup of the superembedding approach.

In particular, such a superembedding description, leading to eq. (4.31) or to $\tilde{q}'F_{ab} = \mathcal{O}(\Upsilon) + \mathcal{O}(\mathcal{W})$, cannot be used to construct an action by superembedding based method of [30], starting from the candidate Wess-Zumino term of eqs. (2.21), (2.18). Hence, this procedure which applies for all known branes, fails in the case of hypothetical Q7-brane. This suggests to conclude that a supersymmetric and κ-symmetric Q7-brane action does not exist.

One can also solve (4.31) by setting $q' = 0$, but this would imply that two gauge fields are of the same type and actually, when $\Upsilon = 0 = \mathcal{W}$, would result in their coincidence. This does not correspond to a Q7-brane, as far as the corresponding charge matrix Q is degenerate, but rather to a (S)D7-brane itself (see eqs. (2.22) and (2.23)).

5 Summary, conclusions and discussion.

The main aim of our present study was to search for possible description of hypothetical Q7-branes dynamics in the frame of superembedding approach.

We begin by the analysis of candidate Wess-Zumino (WZ) term which confirmed the suggestion of previous studies in purely bosonic limit [2] on that the Q7-brane, if existed, would carry out two worldvolume gauge fields on its worldvolume as well as on the general structure of the candidate WZ term constructed with these two gauge fields. This could be used to construct the complete action in an algorithmic manner if one lifts this WZ term to a 9-form in the 7-brane worldvolume superspace with 8 bosonic and 16 fermionic 'directions', $W^{(8|16)}$. This, in its turn, would be possible if it existed the possibility to describe two different gauge potentials by superfields on this worldvolume superspace.

Motivated by this observation, we studied the possibilities to describe two independent dynamical gauge fields on the worldvolume of a type IIB 7-brane. If consistent, this would be the description of the Q7-brane of [2]. However, the result of our study is negative.

The 7-brane superembedding equation is off-shell. Imposing the constraints on the generalized field strength $F_2 = dA - q\hat{B}_2 - q'\hat{C}_2$ of the worldvolume gauge superfield (super-1-form A) leads to the equations of motion for SD7-brane related to D7-brane by SL(2) transformation (called '(p,q) 7-branes' in [2]). The case of D7-brane itself corresponds to $(q, q') = (1, 0)$, and, in the light of existence of the SL(2) covariant formalism [19], it is sufficient to use this charge configuration as a reference point.

Then the natural hypothesis, also supported by the results on the bosonic Q7-brane action and inclusion of fermions in the linear approximation, is that Q7-brane might be described by a SD7-brane carrying an additional gauge field on its worldvolume. If this were the case, imposing the superspace constraints of an additional SYM super-1-form leaving on a D7-brane worldvolume we would obtain a consistent system of dynamical equations which would be the equations of motion for the Q7-brane.

We have done this in section 4, and the result of this analysis has been negative. Namely, the consistency of the second gauge field constraints requires that either these are of the same type as the original one, and then the second field strength coincide with the first one, $G_{ab} = F_{ab}$, or, for the different second field strength (including different combination of RR and NS-NS fields in its definition), it results in $F_{ab} = 0$. This relation can characterize a field configuration rather than equations of motion of a dynamical brane with two gauge fields.

This suggests that, probably, Q7-brane does not exists as supersymmetric dynamical system, but there exists only the Q7-brane BPS state described by a particular solution of supergravity equations [1, 3] and also of some non-supersymmetric system of worldvolume equations. If this is the case, then, when imposing the requirement of supersymmetry (as we do when develop superembedding approach, by its construction [8]–[18]) this supersymmetric solution describes the only field configuration which is allowed by consistency of the system of manifestly supersymmetric equations, thus resulting in the solution rather than in a system of dynamical equations (as it would be the case if we were considering a true dynamical superbrane). Actually, a search for particular BPS solution of such a type might be an interesting new application of the superembedding approach.

One might also think that more general setup is needed to conclude definitely about non-existence of Q7-brane. First idea might be to try to introduce, instead of the second gauge field, a 5-form potential the generalized field strength of which, $\mathcal{F}_{abcdef}$, would be dual to the one of the second gauge field strength, G_{ab}, on the mass shell. However, one can easily check, using the natural form of the $\tilde{\mathcal{F}}_6$ superspace constraints

$$\tilde{\mathcal{F}}_6 := dA_5 - \hat{C}_6 - \mathcal{F}_2 \wedge \hat{C}_4 - \frac{1}{2}\mathcal{F}_2 \wedge \mathcal{F}_2 \wedge C_2 = \frac{1}{6!} e^{a_6} \wedge \ldots \wedge e^{a_1} F_{a_1 \ldots a_6} \, , \tag{5.1}$$

that, when considered on the D7-brane worldvolume superspace, the Bianchi identities for this constraint imply a kind of *duality to the 1-form gauge field strength entering the superembedding supermultiplet*,

$$e^{\hat{\Phi}} \sqrt{|\eta + F|} (\eta - F)^{-1}{}_a{}^b \, F_{bb_1 \ldots b_5} = \frac{1}{2} \varepsilon_{ab_1 \ldots b_5 cd} F^{cd} + \mathcal{O}(FF) \, , \tag{5.2}$$

and not to some second independent gauge fields strength, as wanted.

One might also think on that our constraints for the two gauge fields on 7-brane worldvolume, (4.1) and (4.5), are too strong, and search for more general ones. However, we notice that i) already the inclusion of additional superfields Υ and $\mathcal{W}^q$ in (4.5) creates a problem as these do not have any place in the Q7-brane picture as conjectured in [2], and also that ii) these constraints are consistent on linearized level leading to a correct free field equations.

Then one might think that a possibility to describe Q7-brane might appear when one rejects imposing the superembedding equation.

Although our study does not give a definite answer on this query, and so a possibility of radical reformulation of superembedding approach to incorporate new exotic superbranes is not ruled out, we do not expect this to be the case for Q7-branes.[22] The reasons are that all

[22] A modification of superembedding equation does occur in the 'minus one quantization' picture of [38], where the non-Alebian structure of coincident Dp-brane is described by the boundary fermions. But in this case the boundary fermions are included in the set of worldvolume superspace coordinates, but not in the set of tangent superspace ones, so that, similarly to superfield description of spinning string, the number of fermionic dimensions of the worldsheet superspace exceeds the one of the target space. This is not the case for superembedding description of 1/2 BPS superbranes, where the target superspace has twice more fermionic dimensions that the worldvolume superspace.

the known dynamical branes (1/2 BPS superbranes) allow for the description by worldvolume superspace whose embedding is characterized by the superembedding equation (either alone, or together with other constraints), and also that in the case of 7-branes the superembedding equation is off-shell, i.e. it is not strong enough even to produce equations of motion.

Finally, one could propose to assume that Q7-brane might be a brane which cannot be described by the superembedding approach. But, again, no examples of such a situation are known and no reason are seen for a dynamical brane to do not allow for an on-shell superfield superembedding description.

The above arguments suggest (although, of course, do not prove) the conjecture that Q7-brane of [1, 2] is not a dynamical brane, but just a particular BPS configuration of the system of two different SD7-branes; Q7-branes can be described by the supersymmetric solution [1] of type IIB supergravity equations, but neither κ-symmetric and supersymmetric action, nor supersymmetric equations of motion may be associated to them. If this is the case, the superembedding description of such a BPS state is also given by a configuration of worldvolume superfields, rather than by (super)fields obeying dynamical equations of motion, and this is exactly what we have observed in our study of the superembedding approach to 7-branes.

Our study and the above discussion might provide a suggestion for the old standing problem of that the commonly accepted action for coincident Dp-branes [34] does not possess neither supersymmetry nor Lorentz symmetry (see footnote 6 for references on recent progress). The situation with Q7-brane seems to be similar: one can write down a bosonic action [2] (and in this case, in contrast to [34], at least the Lorentz SO(1,9) symmetry is respected), but our study suggests that supersymmetric generalization of the equations of motion fails and so would fail an attempt of supersymmetric generalization of that action. It may be that in both cases we deal with an action for a dynamical system of several branes (two different SD7-brane in Q7-case or N Dp-branes in the 'dielectric brane' action of [34]) which is not supersymmetric.

How it could be if, speaking about multi-brane system, one implies an action obtained in some way from a sum of two or more SD7-brane actions, which are supersymmetric by construction? The possible answer is that the sum of, say, two SD7-brane actions *lost* supersymmetry when one requires the existence of an intersection or, more generally, of some set of common points of two worldvolumes (see [33]).

To clarify this issue, it seems convenient to speak about intersections and in terms of local supersymmetry preserved by bosonic brane actions [50, 51] which represents the κ-symmetry of the original superbranes. On the intersection (or on a set of common points) we, roughly speaking, should impose on the local supersymmetry parameter the condition that it vanishes by the two projectors corresponding to the κ-symmetries of two intersecting branes. But this system of two equation have nontrivial solutions only for definite angles of the intersection [49]. On the other hand, considering the action principle or effective equations of motion for a bound state (for a composed system), it does not look proper to fix the angle of the intersection. Thus we have an action of, say, string ending on a Dp-brane in which the supersymmetry is broken on the intersection; however, there exists a ground state solution where this broken supersymmetry is restored [33].[23]

Now, in the superembedding approach we are searching for supersymmetric field configurations (usually, for the supersymmetric equations of motion). What we have to find for a non-supersymmetric system with a particular supersymmetric solution? - All the conditions for the fields which guarantee that the field configuration is supersymmetric. Thus we have to arrive at a particular supersymmetric field configuration rather than at a set of supersymmetric equations. This is just what happens in the case of Q7-brane as described by (S)D7-brane carrying an additional worldvolume gauge field: the preservation of 1/2 of the supersymmetry in the presence of the second, different gauge field implies vanishing of the 'original' (S)D7-brane gauge fields (modulo scalar fields and fermions, but this does not change the conclusion); this is characteristic for a solution rather than for dynamical equations describing a dynamical object, and this is just the solution which possesses supersymmetry.

Of course, it is always hard to prove non-existence of some object, and our results cannot be considered as a proof. However, on one hand we find the above arguments against the existence of a Q7-brane as a dynamical supersymmetric object convincing and, on the other hand, we hope that, if it finally were found that some exotic possibility for constructing the supersymmetric action and/or equations of motion for Q7-branes happened to exist, our discussion in this paper would be helpful in search for such an exotic construction, e.g. for an exotic modification of the superembedding approach.

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[23] In [54] an introduction of additional degrees of freedom on the intersection was proposed to take care of this 'anomaly'. Such additional degrees of freedom might come from supergravity part of the superbrane-supergravity dynamical system (see [51] and refs therein for the complete but gauge fixed Lagrangian description of such a system). Such a point of view seems to be in agreement with resent description of coincident D-branes by boundary fermions at the ends of string [38]. Indeed, to reproduce the non-Abelian structure of the coincident Dp-brane actions form the approach of [38] the quantization of boundary fermions is to be done. In such a quantization the Myers action is reproduced [38] and the Lorentz covariance is lost. To obtain the completely covariant and supersymmetric result one (presumably) needs to carry out the complete quantization of the string model with boundary fermions, and not just the boundary fermion sector. However, the complete quantization of open superstring should also reproduce the supergravity fields (from the closed string sector). Their influence might then come in the form of the above mentioned additional degrees of freedom in the effective low energy multiple brane action. The further analysis of these issues goes beyond the score of this paper.

A Spinors in ten and eight dimensions

In $D = 10$ we use the Majorana-Weyl spinors and real symmetric sigma matrices, $\sigma^{\underline{a}}_{\alpha\beta} = \sigma^{\underline{a}}_{\beta\alpha}$ and $\tilde{\sigma}_{\underline{a}}^{\alpha\beta} = \tilde{\sigma}_{\underline{a}}^{\beta\alpha}$, which obey

$$\sigma^{\underline{a}}\tilde{\sigma}^{\underline{b}} + \sigma^{\underline{b}}\tilde{\sigma}^{\underline{a}} = \eta^{\underline{ab}} \, , \qquad \tilde{\sigma}^{\underline{a}}\sigma^{\underline{b}} + \tilde{\sigma}^{\underline{b}}\sigma^{\underline{a}} = \eta^{\underline{ab}} \, , \qquad \eta^{\underline{ab}} = \mathrm{diag}(1, -1, \dots, -1) \, , \qquad \underline{a}\, , \underline{b} = 0, 1, \dots, 9 \, , \tag{A.1}$$

as well as the famous $D = 10$ identity

$$\sigma_{\underline{a}\,(\alpha\beta}\sigma^{\underline{a}}{}_{\gamma)\delta} = 0 \, , \qquad \tilde{\sigma}^{\underline{a}\,(\alpha\beta}\tilde{\sigma}_{\underline{a}}{}^{\gamma)\delta} = 0 \, . \tag{A.2}$$

We define $\sigma^{\underline{ab}} = \sigma^{[\underline{a}}\tilde{\sigma}^{\underline{b}]} = \frac{1}{2}(\sigma^{\underline{a}}\tilde{\sigma}^{\underline{b}} - \sigma^{\underline{a}}\tilde{\sigma}^{\underline{b}})$ as well as $\tilde{\sigma}^{\underline{ab}} = \tilde{\sigma}^{[\underline{a}}\sigma^{\underline{b}]}$, $\sigma^{\underline{abc}} = \sigma^{[\underline{a}}\tilde{\sigma}^{\underline{b}}\sigma^{\underline{c}]}$ etc. Matrices $\sigma_{\underline{a}}$, $\sigma_{\underline{a}_1\cdots\underline{a}_5}$ and $\tilde{\sigma}_{\underline{a}}$, $\tilde{\sigma}_{\underline{a}_1\cdots\underline{a}_5}$ are symmetric, $\sigma_{\underline{a}_1\underline{a}_2\underline{a}_3}$ and $\tilde{\sigma}_{\underline{a}_1\underline{a}_2\underline{a}_3}$ are antisymmetric, furthermore, $\sigma_{\underline{a}_1\cdots\underline{a}_5}$ is self-dual while $\tilde{\sigma}_{\underline{a}_1\cdots\underline{a}_5}$ is anti-self dual.

In eight dimensions, $d = 1 + 7$, one can define 16-component pseudo-real (pseudo-Majorana) spinors

$$(\chi^q)^* = \gamma^0_{qp}\chi^p \, , \qquad q, p = 1, \dots, 16 \, . \tag{A.3}$$

The charge conjugation matrix is symmetric and can be identified with the the unity matrix δ_{qp}, the seven gamma matrices γ^a_{qp} are symmetric and pseudo real,

$$\gamma^a_{qp} = \gamma^a_{pq} \, , \qquad (\gamma^a)^* = \gamma^0\gamma^a\gamma^0 \, , \qquad a = 0\, , 1\, , \dots\, , 7\, , \qquad q, p = 1, \dots, 16 \, . \tag{A.4}$$

This is also the case for their product which we call γ^9_{qp},

$$\gamma^9_{qp} := \gamma^0\gamma^1 \dots \gamma^7 = \gamma^9_{pq} \, , \qquad (\gamma^9)^* = \gamma^0\gamma^9\gamma^0 = -\gamma^9 \, . \tag{A.5}$$

This can be used to construct two chiral projectors $\frac{1}{2}(\delta \pm i\gamma^9)$

$$\delta_{qp} = \frac{1}{2}(\delta + i\gamma^9)_{qp} + \frac{1}{2}(\delta - i\gamma^9)_{qp} \, , \qquad \begin{cases} \frac{1}{2}(\delta + i\gamma^9)\frac{1}{2}(\delta + i\gamma^9) = \frac{1}{2}(\delta + i\gamma^9) \, , \\ \frac{1}{2}(\delta - i\gamma^9)\frac{1}{2}(\delta - i\gamma^9) = \frac{1}{2}(\delta - i\gamma^9) \\ \frac{1}{2}(\delta + i\gamma^9)\frac{1}{2}(\delta - i\gamma^9) = 0 \end{cases} \, , \tag{A.6}$$
$$(\delta + i\gamma^9)^* = (\delta + i\gamma^9) = \gamma^0(\delta - i\gamma^9)\gamma^0 \, .$$

Let us notice the $d = 8$ gamma-metrix identity

$$\begin{aligned} \gamma_{b\,(q_1 q_2}\,\gamma^b_{q_3)p} &= \delta_{(q_1 q_2}\,\delta_{q_3)p} + \gamma^9_{(q_1 q_2}\,\gamma^9_{q_3)p} = \\ &= \frac{1}{2}(\delta - i\gamma^9)_{(q_1 q_2}(\delta + i\gamma^9)_{q_3)p} + \frac{1}{2}(\delta + i\gamma^9)_{(q_1 q_2}(\delta - i\gamma^9)_{q_3)p} \, . \end{aligned} \tag{A.7}$$

which can be derived e.g. from the $D = 10$ identity (A.2). Useful consequences of (A.7) are

$$((\delta + i\gamma^9)\gamma_b)_{(q_1|q}\,((\delta + i\gamma^9)\gamma^b)_{|q_2)p} = (\delta + i\gamma^9)_{q_1 q_2}\,(\delta - i\gamma^9)_{qp} \, , \tag{A.8}$$
$$((\delta - i\gamma^9)\gamma_b)_{(q_1|q}\,((\delta - i\gamma^9)\gamma^b)_{|q_2)p} = (\delta - i\gamma^9)_{q_1 q_2}\,(\delta + i\gamma^9)_{qp} \, . \tag{A.9}$$

As it is well known, in SO(1,9) covariant notation ($D = 10$) the basis of symmetric 16×16 matrices is provided by 10 $\sigma^{\underline{a}}_{\alpha\beta}$ and 126 (five index self-dual) $\sigma^{\underline{a}_1\cdots\underline{a}_5}_{\alpha\beta} := \frac{1}{5!}\varepsilon^{\underline{a}_1\cdots\underline{a}_5\underline{b}_1\cdots\underline{b}_5}\sigma_{\underline{b}_1\cdots\underline{b}_5\alpha\beta}$, while the basis of the antisymmetric matrices is provided by 120 $\sigma^{\underline{a}_1\cdots\underline{a}_3}_{\alpha\beta}$. In the SO(1,7) covariant notation ($d = 8$) this corresponds to the following bases of the symmetric and antisymmetric matrices:

$$\text{SYMM}: \ \delta_{qp}\,,\ \gamma^9_{qp}\,,\ \gamma^a_{qp}\,,\ (\gamma^{abc}\gamma^9)_{qp}\,,\ (\gamma^{abcd})_{qp}\,, \quad 1+1+8+56+70 = 136\,, \tag{A.10}$$

$$\text{AntiSYMM}: \ (\gamma^a\gamma^9)_{qp}\,,\ \gamma^{ab}_{qp}\,,\ (\gamma^{ab}\gamma^9)_{qp}\,,\ (\gamma^{abc})_{qp}\,, \quad 8+28+28+56 = 120\,. \tag{A.11}$$

Notice that $(\gamma^{abcd}\gamma^9)_{qp} = \frac{1}{4!}\varepsilon^{abcda'b'c'd'}(\gamma_{a'b'c'd'})_{pq}$ and, hence, is not independent.

B Type IIB supergravity constraints and their consequences

Denoting the fermionic vielbein of type IIB superspace as in (1.1),

$$\mathcal{E}^{\underline{\beta}} := \mathcal{E}^{\beta i} := (E^{\beta 1}\,,\ E^{\beta 2})\,, \tag{B.1}$$

one can write the bosonic torsion constraint in the form of (1.2),

$$T^{\underline{a}} := DE^{\underline{a}} = -i\mathcal{E}\wedge\sigma^{\underline{a}}\mathcal{E}\,, \qquad \sigma^{\underline{a}}_{\underline{\alpha\beta}} := \sigma^{\underline{a}}_{\alpha\beta}\delta_{ij}\,. \tag{B.2}$$

The fermionic torsion forms of type IIB superspace $T^{\underline{\beta}} := D\mathcal{E}^{\underline{\beta}} = (T^{\beta 1}\,,\ T^{\beta 2})$ are

$$\begin{aligned} T^{\alpha 1} &= -E^{\alpha 1}\wedge E^{\beta 1}\nabla_{\beta 1}e^{-\Phi} + \frac{1}{2}E^1\sigma^{\underline{a}}\wedge E^1\,\tilde{\sigma}^{\alpha\beta}_{\underline{a}}\nabla_{\beta 1}e^{-\Phi} + \\ &\quad + E^{\underline{a}}\wedge\mathcal{E}^{\underline{\beta}}T_{\underline{\beta}\underline{a}}{}^{\alpha 1} + \frac{1}{2}E^{\underline{b}}\wedge E^{\underline{a}}T_{\underline{ab}}{}^{\alpha 1}\,, \end{aligned} \tag{B.3}$$

$$\begin{aligned} T^{\alpha 2} &= -E^{\alpha 2}\wedge E^{\beta 2}\nabla_{\beta 2}e^{-\Phi} + \frac{1}{2}E^2\sigma^{\underline{a}}\wedge E^2\,\tilde{\sigma}^{\alpha\beta}_{\underline{a}}\nabla_{\beta 2}e^{-\Phi} + \\ &\quad + E^{\underline{a}}\wedge\mathcal{E}^{\underline{\beta}}T_{\underline{\beta}\underline{a}}{}^{\alpha 2} + \frac{1}{2}E^{\underline{b}}\wedge E^{\underline{a}}T_{\underline{ab}}{}^{\alpha 2}\,, \end{aligned} \tag{B.4}$$

where

$$\begin{aligned} T_{\underline{\alpha b}}{}^{\underline{\gamma}} &=: -t_{\underline{b\alpha}}{}^{\underline{\gamma}} = -\frac{1}{8}H_{\underline{bcd}}\,(\sigma^{\underline{cd}}\tau_3)_{\underline{\alpha}}{}^{\underline{\gamma}} + \sum_{n=0}^{4}(\sigma_{\underline{b}}\tilde{R\!\!\!/}^{(2n+1)}\tau_1(\tau_3)^n)_{\underline{\alpha}}{}^{\underline{\gamma}} = \\ &= -\frac{1}{8}\left(H_{\underline{bcd}}\,\sigma^{\underline{cd}}\tau_3 - \sigma_{\underline{b}}\tilde{R\!\!\!/}^{(1)}i\tau_2 + \sigma_{\underline{b}}\tilde{R\!\!\!/}^{(3)}\tau_1 - \tfrac{1}{2}\sigma_{\underline{b}}\tilde{R\!\!\!/}^{(5)}i\tau_2\right)_{\underline{\alpha}}{}^{\underline{\gamma}}\,, \end{aligned} \tag{B.5}$$

$$\tilde{R\!\!\!/}^{(2n+1)} := \frac{1}{(2n+1)!}R_{\underline{a}_1\cdots\underline{a}_{2n+1}}\tilde{\sigma}^{\underline{a}_1\cdots\underline{a}_{2n+1}}\,. \tag{B.6}$$

Other useful equations are

$$D_{\underline{\hat{\beta}}}D_{\underline{\hat{\gamma}}}e^{-\Phi} = i\sigma^{\underline{a}}_{\underline{\hat{\beta}\hat{\gamma}}}D_{\underline{a}}e^{-\Phi} - i(\tilde{R\!\!\!/}^{(1)}i\tau_2)_{\underline{\hat{\beta}\hat{\gamma}}} + \frac{i}{2}(\tilde{R\!\!\!/}^{(3)}\tau_1)_{\underline{\hat{\beta}\hat{\gamma}}} + \frac{i}{2}(\tilde{H\!\!\!/}^{(3)}\tau_3)_{\underline{\hat{\beta}\hat{\gamma}}}\,. \tag{B.7}$$

and

$$T_{\underline{\beta\gamma}}{}^{\underline{\alpha}}D_{\underline{\alpha}} = \begin{pmatrix} -2\left(\delta_{(\beta}{}^{\alpha}D_{\gamma)1}e^{-\Phi} - \frac{1}{2}\sigma^{\underline{a}}_{\beta\gamma}\tilde{\sigma}^{\alpha\delta}_{\underline{a}}D_{\delta 1}e^{-\Phi}\right)D_{\alpha 1} & 0 \\ 0 & -2\left(\delta_{(\beta}{}^{\alpha}D_{\gamma)2}e^{-\Phi} - \frac{1}{2}\sigma^{\underline{a}}_{\beta\gamma}\tilde{\sigma}^{\alpha\delta}_{\underline{a}}\,D_{\delta 2}e^{-\Phi}\right)D_{\alpha 2} \end{pmatrix}. \tag{B.8}$$

The constraints for the NS-NS three form field strength are

$$H_3 = -iE^{\underline{a}} \wedge (E^1 \wedge \sigma_{\underline{a}} E^1 - E^2 \wedge \sigma_{\underline{a}} E^2) + \frac{1}{3!} E^{\underline{c}_3} \wedge E^{\underline{c}_2} \wedge E^{\underline{c}_1} H_{\underline{c}_1 \underline{c}_2 \underline{c}_3} \, . \tag{B.9}$$

The RR field strengths obey the constraints

$$\begin{aligned} R_{2n+1} = 2ie^{-\Phi} E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}^{(2n-1)}_{\alpha\beta} - e^{-\Phi} \left(E^2 \wedge \bar{\sigma}^{(2n)} \nabla_1 \Phi - (-)^n E^1 \wedge \bar{\sigma}^{(2n)} \nabla_2 \Phi \right) + \\ + \frac{1}{(2n+1)!} E^{\underline{a}_{2n+1}} \wedge \ldots \wedge E^{\underline{a}_1} R_{\underline{a}_1 \cdots \, \underline{a}_{2n+1}} \, , \end{aligned} \tag{B.10}$$

where

$$\bar{\sigma}^{(2n+1)}_{\alpha\beta} := \tfrac{1}{(2n+1)!} E^{\underline{a}_{2n+1}} \wedge \ldots \wedge E^{\underline{a}_1} (\sigma_{\underline{a}_1 \cdots \, \underline{a}_{2n+1}})_{\alpha\beta} \, . \tag{B.11}$$

Eq. (B.10) includes, as particular cases,

$$R_1 = E^2 \, \nabla_1 e^{-\Phi} - E^1 \, \nabla_2 e^{-\Phi} + E^{\underline{c}} R_{\underline{c}} \, , \tag{B.12}$$

$$\begin{aligned} R_3 &= 2ie^{-\Phi} E^2 \wedge E^1 \wedge \sigma^{(1)} + E^2 \wedge \sigma^{(2)} \, \nabla_1 e^{-\Phi} + E^1 \wedge \sigma^{(2)} \, \nabla_2 e^{-\Phi} + \\ &\quad + \frac{1}{3!} E^{\underline{c}_3} \wedge E^{\underline{c}_2} \wedge E^{\underline{c}_1} R_{\underline{c}_1 \underline{c}_2 \underline{c}_3} \, , \end{aligned} \tag{B.13}$$

$$\begin{aligned} R_5 &= 2ie^{-\Phi} E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}^{(3)}_{\alpha\beta} + \left(E^2 \wedge \bar{\sigma}^{(4)} \nabla_1 e^{-\Phi} - E^1 \wedge \bar{\sigma}^{(4)} \nabla_2 e^{-\Phi} \right) + \\ &\quad + \frac{1}{5!} E^{\underline{a}_5} \wedge \ldots \wedge E^{\underline{a}_1} R_{\underline{a}_1 \cdots \, \underline{a}_5} \, , \end{aligned} \tag{B.14}$$

as well as the constraints for the super-field strength of the higher forms

$$\begin{aligned} R_7 &= 2ie^{-\Phi} E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}^{(5)}_{\alpha\beta} - e^{-\Phi} \left(E^2 \wedge \bar{\sigma}^{(6)} \nabla_1 \Phi + E^1 \wedge \bar{\sigma}^{(6)} \nabla_2 \Phi \right) + \\ &\quad + \frac{1}{7!} E^{\underline{a}_7} \wedge \ldots \wedge E^{\underline{a}_1} R_{\underline{a}_1 \cdots \, \underline{a}_7} \, , \end{aligned} \tag{B.15}$$

$$\begin{aligned} R_9 &= 2ie^{-\Phi} E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}^{(7)}_{\alpha\beta} - e^{-\Phi} \left(E^2 \wedge \bar{\sigma}^{(8)} \nabla_1 \Phi - (-)^n E^1 \wedge \bar{\sigma}^{(8)} \nabla_2 \Phi \right) + \\ &\quad + \frac{1}{9!} E^{\underline{a}_9} \wedge \ldots \wedge E^{\underline{a}_1} R_{\underline{a}_1 \cdots \, \underline{a}_9} \, , \end{aligned} \tag{B.16}$$

whose higher-dimensional purely bosonic tensor parts are dual to the bosonic field strengths of the lower forms

$$R_{\underline{a}_1 \ldots \underline{a}_{9-2n}} = \frac{(-)^n}{(2n+1)!} \, \varepsilon_{\, \underline{a}_1 \ldots \underline{a}_{9-2n} \underline{b}_1 \ldots \underline{b}_{2n+1}} R^{\underline{b}_1 \ldots \underline{b}_{2n+1}} \qquad : \tag{B.17}$$

$$R_{\underline{a}_1 \ldots \underline{a}_9} = \varepsilon_{\, \underline{a}_1 \ldots \underline{a}_9 \underline{b}} R^{\underline{b}} \, , \qquad R_{a_1 \ldots a_7} = -\frac{1}{3!} \varepsilon_{\, a_1 \ldots a_7 b_1 \ldots b_3} R^{b_1 \ldots b_3} \, , \tag{B.18}$$

$$R_{a_1 \ldots a_5} = (*R)_{a_1 \ldots a_5} := \frac{1}{5!} \varepsilon_{\, a_1 \ldots a_5 b_1 \ldots b_5} R^{b_1 \ldots b_5} \tag{B.19}$$

It is also convenient to define the ten-form potential C_{10} with the field strength $R_{11} = dC_{10} - H_3 \wedge C_8$ which is nonzero due to the presence of the fermionic directions only,

$$\begin{aligned} R_{11} &= 2ie^{-\Phi} E^{\alpha 2} \wedge E^{\beta 1} \wedge \bar{\sigma}^{(9)}_{\alpha\beta} - e^{-\Phi} \left(E^2 \wedge \bar{\sigma}^{(10)} \nabla_1 \Phi + E^1 \wedge \bar{\sigma}^{(10)} \nabla_2 \Phi \right) = \\ &= 2ie^{-\Phi} E^{\wedge 9}_{\underline{a}} \sigma^{\underline{a}}_{\alpha\beta} \wedge E^{\alpha 2} \wedge E^{\beta 1} - e^{-\Phi} E^{\wedge 10} \wedge \left(E^2 \nabla_1 \Phi + E^1 \nabla_2 \Phi \right) \, . \end{aligned} \tag{B.20}$$

C $\frac{\mathrm{SO}(1,9)}{\mathrm{SO}(1,7)\times\mathrm{SO}(2)}$ moving frame variables (also called Lorentz harmonics)

The moving frame variables (Lorentz harmonics) suitable for the description of the D7-branes and other 7-branes can be defined as $\mathrm{SO}(1,8)\times\mathrm{SO}(2)$ covariant blocks of the $\mathrm{SO}(1,10)$ valued matrix

$$U_{\underline{a}}^{(\underline{b})} := \left(u_{\underline{a}}{}^{b}\, ,\, \frac{1}{2}(u_{\underline{a}}{}^{z} + \bar{u}_{\underline{a}}{}^{\bar{z}})\, ,\, \frac{1}{2i}(u_{\underline{a}}{}^{z} - \bar{u}_{\underline{a}}{}^{\bar{z}}) \right) \quad \in \quad \mathrm{SO}(1,9)\, , \tag{C.1}$$

$$u_{\underline{a}}{}^{z} \;=\; u_{\underline{a}}{}^{8} + i u_{\underline{a}}{}^{9} = (\bar{u}_{\underline{a}}{}^{\bar{z}})^{*}\, , \qquad \underline{a},\underline{b},\underline{c} = 0,\, 1\, , \ldots\, , 9\, , \qquad a,b,c = 0,\, 1\, , \ldots\, , 7\, , \tag{C.2}$$

where $u_{\underline{a}}{}^{z} = u_{\underline{a}}{}^{8} + i u_{\underline{a}}{}^{9} = (\bar{u}_{\underline{a}}{}^{\bar{z}})^{*}$. The condition (C.1) implies

$$U^{T}\eta U = \eta \quad \Leftrightarrow \quad \begin{cases} u^{\underline{c}a} u_{\underline{c}}{}^{b} = \eta^{ab}\, , & u^{\underline{a}a} u_{\underline{a}}{}^{z} = 0 = u^{\underline{a}a}\bar{u}_{\underline{a}}{}^{\bar{z}}\, , \\ u^{\underline{a}z} u_{\underline{a}}{}^{z} = 0 = \bar{u}^{\underline{a}\bar{z}}\bar{u}_{\underline{a}}{}^{\bar{z}}\, , & \\ u^{\underline{a}z}\bar{u}_{\underline{a}}{}^{\bar{z}} = -2\, , & \end{cases} \tag{C.3}$$

as well as $U\eta U^{T} = \eta$, which is equivalent to the following 'unity decomposition'

$$U\eta U^{T} = \eta \quad \Leftrightarrow \qquad \delta_{\underline{a}}{}^{\underline{b}} = U_{\underline{a}}^{(\underline{c})} U_{(\underline{c})}{}^{\underline{b}} = u_{\underline{a}}{}^{c} u_{c}^{\underline{b}} - \frac{1}{2} u_{\underline{a}}{}^{z}\bar{u}^{\underline{b}\bar{z}} - \frac{1}{2}\bar{u}_{\underline{a}}{}^{\bar{z}}\, u^{\underline{b}\, z}\, . \tag{C.4}$$

The pseudo-real *spinor moving frame* matrix $V_{\alpha}{}^{q}$ obeys

$$(V_{\alpha}{}^{q})^{*} = \gamma^{0}_{qp} V_{\alpha}{}^{p}\, , \qquad V\sigma^{\underline{a}}V^{T} = \sigma^{(\underline{b})} U_{(\underline{b})}{}^{\underline{a}}\, , \qquad V^{T}\sigma^{(\underline{a})}V = \sigma^{\underline{b}} U_{\underline{b}}^{(\underline{a})}\, . \tag{C.5}$$

This implies that it is $\mathrm{Spin}(1,9)$ group valued and allows to refer on it as on square root of the moving frame variables.

The spinor moving frame matrix $V_{\alpha}{}^{q}$ carries one $\mathrm{SO}(1,9)$ and one $\mathrm{SO}(1,7)$ spinor index and, hence, can be used as a 'bridge' (in terminology of [46]) to convert the 16 component D=10 Majorana-Weyl spinor (with the index denoted by a Greek symbol) into a 16 component pseudo-real $\mathrm{SO}(1,7)$ spinor (the index of which we denote by p or q). For instance, the pull-backs of the fermionic supervielbein one forms to the worldvolume, $\hat{E}^{\alpha 1} = d\hat{Z}^{\underline{M}}(\xi)E_{\underline{M}}{}^{\alpha 1}(\hat{Z})$, $\hat{E}^{\alpha 2} = d\hat{Z}^{\underline{M}}(\xi)E_{\underline{M}}{}^{\alpha 2}(\hat{Z})$, which carry the D=10 MW spinorial indices, can be established to be in one-to-one correspondence with the pseudo-real one-forms

$$\hat{E}^{q1} := \hat{E}^{\alpha 1}V_{\alpha}{}^{q} = d\hat{Z}^{\underline{M}}(\xi)E_{\underline{M}}{}^{\alpha 1}(\hat{Z}(\xi))V_{\alpha}{}^{q}(\xi)\, , \qquad (\hat{E}^{q1})^{*} = \gamma^{0}_{qp}\hat{E}^{p1}\, , \tag{C.6}$$

$$\hat{E}^{q2} := \hat{E}^{\alpha 2}V_{\alpha}{}^{q} = d\hat{Z}^{\underline{M}}(\xi)E_{\underline{M}}{}^{\alpha 2}(\hat{Z}(\xi))V_{\alpha}{}^{q}(\xi)\, , \qquad (\hat{E}^{q2})^{*} = \gamma^{0}_{qp}\hat{E}^{p2}\, . \tag{C.7}$$

Eqs. (C.5) can be split as

$$\begin{aligned} V_{\alpha}\gamma^{a}V_{\beta} := V_{\alpha}{}^{q}\gamma^{a}_{qp}V_{\beta}{}^{p} &= \sigma^{\underline{b}}_{\alpha\beta}u_{\underline{b}}{}^{a}\, , \\ V_{\alpha}{}^{q}(\delta + i\gamma^{9})_{qp}V_{\beta}{}^{p} &= \sigma^{\underline{b}}_{\alpha\beta}u_{\underline{b}}{}^{z}\, , \qquad V_{\alpha}{}^{q}(\delta - i\gamma^{9})_{qp}V_{\beta}{}^{p} = \sigma^{\underline{b}}_{\alpha\beta}\bar{u}_{\underline{b}}{}^{\bar{z}}\, , \end{aligned} \tag{C.8}$$

$$\begin{aligned} \tilde{\sigma}^{\underline{b}\alpha\beta}u_{\underline{b}}{}^{a} &= V^{\alpha}\gamma^{a}V^{\beta} := V^{\alpha}_{q}\gamma^{a}_{qp}V^{\beta}_{p}\, , \\ \tilde{\sigma}^{\underline{b}\alpha\beta}u_{\underline{b}}{}^{z} &= -V^{\alpha}_{q}(\delta - i\gamma^{9})_{qp}V^{\beta}_{p}\, , \qquad \tilde{\sigma}^{\underline{b}\alpha\beta}\bar{u}_{\underline{b}}{}^{\bar{z}} = -V^{\alpha}_{q}(\delta + i\gamma^{9})_{qp}V^{\beta}_{p}\, , \end{aligned} \tag{C.9}$$

$$V_q \sigma^{\underline{a}} V_p = \gamma^b_{qp} u_b^{\underline{a}} - \frac{1}{2}(\delta + i\gamma^9)_{qp} \bar{u}^{\underline{a}\bar{z}} - \frac{1}{2}(\delta - i\gamma^9)_{qp} u^{\underline{a}\, z} \, . \tag{C.10}$$

To calculate in an easy manner the derivatives of the moving frame variables, one should take into account that the space tangent to a group manifold of a Lie group is isomorphic to the Lie algebra of this Lie group. As far as moving frame variables form the SO(1,9) valued matrix and the spinor moving frame matrix takes its values in Spin(1,9), doubly covering SO(1,9), this allows us to express the derivatives/variations of both moving frame and spinor moving frame variables (vector and spinor Lorentz harmonics) in terms of the same Cartan forms,

$$dU_{\underline{a}}^{(\underline{b})} = U_{\underline{a}(\underline{c})}(U^T dU)^{(\underline{c})(\underline{b})} \, , \qquad \delta U_{\underline{a}}^{(\underline{b})} = U_{\underline{a}(\underline{c})}(U^T \delta U)^{(\underline{c})(\underline{b})} \, , \tag{C.11}$$

$$dV_\alpha{}^q = \frac{1}{4} V_\alpha{}^p (U^T dU)^{(\underline{c})(\underline{b})} \sigma_{(\underline{c})(\underline{b})pq} \, , \qquad \delta V_\alpha{}^q = \frac{1}{4} V_\alpha{}^p (U^T \delta U)^{(\underline{c})(\underline{b})} \sigma_{(\underline{c})(\underline{b})pq} \, . \tag{C.12}$$

When the theory in curved superspace is considered, to keep the local Lorentz invariance preserved, it is convenient to work with the covariant generalizations of the Cartan forms, defined with the use of $SO(1,9)$ covariant derivative $d + w$ instead of the usual derivative d (see [16, 17]). In our case these generalized Cartan forms read

$$\Omega^{az} := u^{\underline{b}a}(du_{\underline{b}}{}^z + w_{\underline{b}}{}^{\underline{c}} u_{\underline{c}}{}^z) =: (udu)^{az} + w^{az} \, , \tag{C.13}$$

$$\bar{\Omega}^{a\bar{z}} := u^{\underline{b}a}(d\bar{u}_{\underline{b}}{}^{\bar{z}} + w_{\underline{b}}{}^{\underline{c}} \bar{u}_{\underline{c}}{}^{\bar{z}}) =: (udu)^{a\bar{z}} + w^{a\bar{z}} \, , \tag{C.14}$$

$$A := \frac{i}{2} u^{\underline{b}z}(d\bar{u}_{\underline{b}}{}^{\bar{z}} + w_{\underline{b}}{}^{\underline{c}} \bar{u}_{\underline{c}}{}^{\bar{z}}) =: \frac{i}{2}((udu)^{z\bar{z}} + w^{z\bar{z}}) \, , \tag{C.15}$$

$$\omega^{ab} := u^{\underline{c}a}(du_{\underline{c}}{}^b + w_{\underline{c}}{}^{\underline{c}'} u_{\underline{c}'}{}^b) =: (udu)^{ab} + w^{ab} \, . \tag{C.16}$$

These definitions can be collected in the expression for the $SO(1,9) \otimes SO(1,7) \otimes SO(2)$ covariant derivatives of the moving frame vectors

$$Du_{\underline{b}}{}^z := du_{\underline{b}}{}^z - iAu_{\underline{b}}{}^z + w_{\underline{b}}{}^{\underline{c}} u_{\underline{c}}{}^z = u_{\underline{b}a} \Omega^{az} \, , \tag{C.17}$$

$$D\bar{u}_{\underline{b}}{}^{\bar{z}} := d\bar{u}_{\underline{b}}{}^{\bar{z}} + iA\bar{u}_{\underline{b}}{}^{\bar{z}} + w_{\underline{b}}{}^{\underline{c}} \bar{u}_{\underline{c}}{}^{\bar{z}} = u_{\underline{b}a} \bar{\Omega}^{a\bar{z}} \, , \tag{C.18}$$

$$Du_{\underline{b}}{}^a := du_{\underline{b}}{}^a - u_{\underline{b}}{}^b \omega_b{}^c + w_{\underline{b}}{}^{\underline{c}} u_{\underline{c}}{}^z = \frac{1}{2} \bar{u}_{\underline{b}}{}^{\bar{z}} \Omega^{az} + \frac{1}{2} u_{\underline{b}}{}^z \bar{\Omega}^{a\bar{z}} \, . \tag{C.19}$$

The covariant version of (C.12) can be, in its turn, written as an expression for the $SO(1,9) \otimes SO(1,7) \otimes SO(2)$ covariant derivatives of the spinor moving frame variables, *i.e* of the spinor moving frame matrix,

$$\begin{aligned} DV_\alpha{}^q &:= dV_\alpha{}^q + \frac{1}{4} w^{\underline{ab}} \sigma_{\underline{ab}\alpha}{}^\beta V_\beta{}^q - \frac{1}{4} \omega^{ab} V_\alpha{}^p \gamma_{abpq} - \frac{1}{2} A V_\alpha{}^p (\gamma_9)_{pq} = \\ &= \frac{1}{4} \, \Omega^{az} \, V_\alpha{}^p (\gamma_a (\delta + i\gamma_9))_{pq} + \frac{1}{4} \, \bar{\Omega}^{a\bar{z}} \, V_\alpha{}^p (\gamma_a (\delta - i\gamma_9))_{pq} \, . \end{aligned} \tag{C.20}$$

Some other relations useful for our study are

$$V^q \tilde{\sigma}_{\underline{a}} V^p = u_{\underline{a}}{}^b \gamma_{b\, qp} + \tfrac{1}{2}(\delta + i\gamma^9)_{qp} u_{\underline{a}}{}^z + \tfrac{1}{2}(\delta - i\gamma^9)_{qp} \bar{u}_{\underline{a}}{}^{\bar{z}} \, . \tag{C.21}$$

(cf. eqs. (C.10), (A.7)),

$$V_q\sigma^{\underline{ab}}V^p = u_a^{\underline{a}}u_b^{\underline{b}}\gamma^{ab}_{qp} + u_a^{[\underline{a}}u^{\underline{b}]z}(\gamma^a(\delta - i\gamma^9))_{qp} + u_a^{[\underline{a}}\bar{u}^{\underline{b}]\bar{z}}(\gamma^a(\delta - i\gamma^9))_{qp} - iu^{z[\underline{a}}\bar{u}^{\underline{b}]\bar{z}}i\gamma^9_{qp},$$
$$V_p\sigma^{\underline{a}}V_{p'}\ V^q\tilde{\sigma}_{\underline{a}}V^{q'} = \gamma^b_{pp'}\gamma_{b\ qq'} + \frac{1}{2}(\delta + i\gamma^9)_{pp'}(\delta + i\gamma^9)_{qq'} + \frac{1}{2}(\delta - i\gamma^9)_{pp'}(\delta - i\gamma^9)_{qq'}. \tag{C.22}$$

Notice the difference of this latter relation with

$$V_p\sigma^{\underline{a}}V_{p'}\ V_q\sigma_{\underline{a}}V_{q'} = \gamma^b_{pp'}\gamma_{b\ qq'} - \frac{1}{2}(\delta + i\gamma^9)_{pp'}(\delta - i\gamma^9)_{qq'} - \frac{1}{2}(\delta - i\gamma^9)_{pp'}(\delta + i\gamma^9)_{qq'}\ . \tag{C.23}$$

This relation can be used to find that the famous $D = 10$ Fierz identity (A.2), $\sigma_{\underline{a}\,(\alpha\beta}\sigma^{\underline{a}}_{\gamma)\delta} = 0$, is represented by eq. (A.7),

$$\gamma_{b(pp'}\gamma^b_{q)q'} = \frac{1}{2}(\delta + i\gamma^9)_{(pp'}(\delta - i\gamma^9)_{q)q'} + \frac{1}{2}(\delta - i\gamma^9)_{(pp'}(\delta + i\gamma^9)_{q)q'}\ . \tag{C.24}$$

To work with fermionic torsion (see eq. (D.2) below) one uses the spin-tensor

$$\begin{aligned}\mathbf{f}_{pp'}{}^{qq'} &:= \delta_{(p}{}^{q}\delta_{p')}{}^{q'} - \frac{1}{2}V_p\sigma^{\underline{a}}V_{p'}\ V^q\tilde{\sigma}_{\underline{a}}V^{q'} = \\ &= \delta_{(p}{}^{q}\delta_{p')}{}^{q'} - \frac{1}{2}\gamma^b_{pp'}\gamma^{qq'}_b - \frac{(\delta + i\gamma^9)_{pp'}}{2}\frac{(\delta + i\gamma^9)_{qq'}}{2} - \frac{(\delta - i\gamma^9)_{pp'}}{2}\frac{(\delta - i\gamma^9)_{qq'}}{2}\ .\end{aligned} \tag{C.25}$$

One can easily check that its trace in 'lower' indices, $\mathbf{f}_{pp}{}^{qq'} := \delta_{pp'}\mathbf{f}_{pp'}{}^{qq'}$ is proportional to unity matrix $\delta^{qq'}$,

$$\mathbf{f}_{pp}{}^{qq'} = -7\delta^{qq'}\ , \tag{C.26}$$

and also that, as a consequence of eqs. (B.3), (B.4) or (B.8) above,

$$\begin{aligned}V_p{}^{\alpha}V_{p'}{}^{\beta}T_{\alpha 1\,\beta 1}{}^{\gamma 1}V_{\gamma}{}^{q} &= -2\mathbf{f}_{pp'}{}^{qq'}V_{q'}{}^{\delta}D_{\delta 1}e^{-\Phi}\ ,\\ V_p{}^{\alpha}V_{p'}{}^{\beta}T_{\alpha 2\,\beta 2}{}^{\gamma 2}V_{\gamma}{}^{q} &= -2\mathbf{f}_{pp'}{}^{qq'}V_{q'}{}^{\delta}D_{\delta 2}e^{-\Phi}\ .\end{aligned} \tag{C.27}$$

Important properties of the $\mathbf{f}_{pp'}{}^{qq'}$ spin-tensor are

$$\begin{aligned}(\delta + i\gamma^9)_{(q_1}{}^{q'}(\delta + i\gamma^9)_{q_2}{}^{p}(\delta + i\gamma^9)_{q_3)}{}^{p'}\mathbf{f}_{pp'}{}^{qq'} &= 0\ ,\\ (\delta - i\gamma^9)_{(q_1}{}^{q'}(\delta - i\gamma^9)_{q_2}{}^{p}(\delta - i\gamma^9)_{q_3)}{}^{p'}\mathbf{f}_{pp'}{}^{qq'} &= 0\ .\end{aligned} \tag{C.28}$$

D Induced worldvolume superspace geometry for 7-branes

The induced worldvolume geometry of the 7-brane worldvolume superspace embedded into the D=10 type IIB superspace in such a way that the supermebedding equation (3.4) and the conventional constraints hold (all these can be collected in eqs. (3.12) and (3.20)), is characterized by the bosonic torsion

$$De^a = -ie^q\wedge e^p(\gamma^a + h\gamma^a h^T)_{pq} + 2ie^b\wedge e^q(h\gamma^a\chi_b)_q + ie^c\wedge e^b\chi_b\gamma^a\chi_c \tag{D.1}$$

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and fermionic torsion two-form,

$$
\begin{aligned}
& De^q = \\
&= \frac{1}{4}e^p \wedge \Omega^{bz}(\gamma_b(\delta + i\gamma^9))_{pq} + \frac{1}{4}e^p \wedge \bar{\Omega}^{b\bar{z}}(\gamma_b(\delta - i\gamma^9))_{pq} - \\
&\quad - e^p \wedge e^{p'} \mathbf{f}_{pp'}{}^{qq'} V_{q'}{}^{\alpha} D_{\alpha 1} e^{-\Phi} + \\
&\quad + e^a \wedge e^p u_a{}^{\underline{b}} (V_p{}^{\alpha} T_{\alpha 1 \underline{b}}{}^{\beta 1} V_\beta{}^q + h_p{}^{p'} V_{p'}{}^{\alpha} T_{\alpha 2 \underline{b}}{}^{\beta 1} V_\beta{}^q) + \frac{1}{4} e^{b'} \wedge e^{a'} u_{a'}{}^{\underline{a}} u_{b'}{}^{\underline{b}} \hat{T}_{\underline{ab}}{}^{\alpha 1} V_\alpha{}^q = \\
&= -\frac{i}{2} e^p \wedge e^{p'} \left((h(\delta + i\gamma^9)\chi_b)_{(p} (\gamma^b(\delta + i\gamma^9))_{p')q} + c.c. - 2i\mathbf{f}_{pp'}{}^{qq'} V_{q'}{}^{\alpha} D_{\alpha 1} e^{-\Phi} \right) - \\
&\quad - \frac{1}{4} e^a \wedge e^p \ \left(K_{ab}{}^z - i\chi_a(\delta + i\gamma^9)\chi_b \right) (\gamma^b(\delta + i\gamma^9))_{pq} - \\
&\quad - \frac{1}{4} e^a \wedge e^p \ \left(\bar{K}_{ab}{}^{\bar{z}} - i\chi_a(\delta - i\gamma^9)\chi_b \right) (\gamma^b(\delta - i\gamma^9))_{pq} + \\
&\quad + e^a \wedge e^p \ u_a{}^{\underline{b}} \left(V_p{}^{\alpha} T_{\alpha 1 \underline{b}}{}^{\beta 1} V_\beta{}^q + h_p{}^{p'} V_{p'}{}^{\alpha} T_{\alpha 2 \underline{b}}{}^{\beta 1} V_\beta{}^q \right) + \frac{1}{4} e^{b'} \wedge e^{a'} u_{a'}{}^{\underline{a}} u_{b'}{}^{\underline{b}} \hat{T}_{\underline{ab}}{}^{\alpha 1} V_\alpha{}^q \ ,
\end{aligned} \tag{D.2}
$$

where $\mathbf{f}_{pp'}{}^{qq'}$ is defined in eq. (C.25).

In particular, the dimension 1/2 fermionic torsion, according to eq. (D.2) reads

$$
\begin{aligned}
T_{pp'}{}^q = -i \Big(& (h(\delta + i\gamma^9)\chi_b)_{(p} (\gamma^b(\delta + i\gamma^9))_{p')q} + (h(\delta - i\gamma^9)\chi_b)_{(p} (\gamma^b(\delta - i\gamma^9))_{p')q} - \\
& - 2i\mathbf{f}_{pp'}{}^{qq'} V_{q'}{}^{\alpha} D_{\alpha 1} e^{-\Phi} \Big) \ ,
\end{aligned} \tag{D.3}
$$

and the dimension 3/2 fermionic torsion spin-tensor is

$$
\begin{aligned}
T_{ap}{}^q = & \frac{1}{4} \ \left(K_{ab}{}^z - i\chi_a(\delta + i\gamma^9)\chi_b \right) (\gamma^b(\delta + i\gamma^9))_{pq} + \\
& + \frac{1}{4} \ \left(\bar{K}_{ab}{}^{\bar{z}} - i\chi_a(\delta - i\gamma^9)\chi_b \right) (\gamma^b(\delta - i\gamma^9))_{pq} - \\
& - u_a{}^{\underline{b}} \left(V_p{}^{\alpha} T_{\alpha 1 \underline{b}}{}^{\beta 1} V_\beta{}^q + h_p{}^{p'} V_{p'}{}^{\alpha} T_{\alpha 2 \underline{b}}{}^{\beta 1} V_\beta{}^q \right) \ .
\end{aligned} \tag{D.4}
$$

Notice that

$$
\begin{aligned}
& T_{ap}{}^q \gamma_b^{pq} = 4 \left(K_{ab}{}^z + \bar{K}_{ab}{}^{\bar{z}} - 2i\chi_a\chi_b \right) - u_a{}^{\underline{c}} \left((V\gamma_b V)_\beta{}^{\alpha} \hat{T}_{\alpha 1 \underline{c}}{}^{\beta 1} + (V\gamma_b h V)_\beta{}^{\alpha} \hat{T}_{\alpha 2 \underline{c}}{}^{\beta 1} \right) , \\
& T_{ap}{}^q (i\gamma_b\gamma^{(9)})^{pq} = -4 \left(K_{ab}{}^z - \bar{K}_{ab}{}^{\bar{z}} + 2\chi_a\gamma^{(9)}\chi_b \right) - iu_a{}^{\underline{c}} \Big((V\gamma_b\gamma^{(9)}V)_\beta{}^{\alpha} \hat{T}_{\alpha 1 \underline{c}}{}^{\beta 1} + \\
& \qquad\qquad + (V\gamma_b\gamma^{(9)} h V)_\beta{}^{\alpha} \hat{T}_{\alpha 2 \underline{c}}{}^{\beta 1} \Big),
\end{aligned} \tag{D.5}
$$

which implies that the D7-brane scalar field equation can be formulated as expressions for the chiral gamma-traces, $T_{ap}{}^q \gamma^a (\delta \pm i\gamma^9)^{pq}$, of the dimension 1 torsion superfield $T_{ap}{}^q$ (see section 3.3.2 for a discussion).

Using (C.28) and the identities $(\delta + i\gamma^9)\gamma_b(\delta + i\gamma^9) = 0 = (\delta - i\gamma^9)\gamma_b(\delta - i\gamma^9)$ one finds

$$
\begin{aligned}
(\delta + i\gamma^9)_{(q_1}{}^q (\delta + i\gamma^9)_{q_2}{}^p (\delta + i\gamma^9)_{q_3)}{}^{p'} T_{pp'}{}^q &= 0 \ , \\
(\delta - i\gamma^9)_{(q_1}{}^q (\delta - i\gamma^9)_{q_2}{}^p (\delta - i\gamma^9)_{q_3)}{}^{p'} T_{pp'}{}^q &= 0 \ .
\end{aligned} \tag{D.6}
$$

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Similarly

$$
\begin{aligned}
(\delta + i\gamma^9)_{q_1}{}^q(\delta + i\gamma^9)_{q_2}{}^p T_{pb}{}^q &= 0\,, \\
(\delta - i\gamma^9)_{q_1}{}^q(\delta - i\gamma^9)_{q_2}{}^p T_{pb}{}^q &= 0\,.
\end{aligned}
\tag{D.7}
$$

Furthermore,

$$
\begin{aligned}
&\frac{1}{2}T_{(q_1q_2}{}^p((\delta + i\gamma^9)_{q_3)p}\Upsilon + c.c.) = i\gamma^b_{(q_1q_2}(h(\delta + i\gamma^9)\chi_b)_{q_3)}\Upsilon + c.c. + \qquad \text{(D.8)} \\
&\quad + \frac{1}{2}\gamma^b_{(q_1q_2}((\delta + i\gamma^9)\gamma_b V\widehat{D_1e^{-\Phi}})_{q_3)}\Upsilon - \frac{1}{2}(\delta + i\gamma^9)_{(q_1q_2}((\delta - i\gamma^9)V\widehat{D_1e^{-\Phi}})_{q_3)}\Upsilon + c.c.\,.
\end{aligned}
$$

Eqs. (D.1), (D.2) follow from the conventional constraints (3.8), (3.18). The fermionic field χ, appearing in the decomposition of the pull-back of the second fermionic superviel-bein, eq. (3.17), is related with the spinorial derivative of the $\mathrm{SO}(1,7)$ spin-tensor $h_p{}^q$ (3.35), appearing in the same eq. (3.17), by

$$
\begin{aligned}
D_{(p}h_{p')}{}^q = & -i(\gamma^a + h\gamma^a h^T)_{pp'}\chi_a{}^q + \frac{i}{2}(h(\delta + i\gamma^9)\chi_a)_{(p}\,(h\gamma^a(\delta + i\gamma^9))_{p')q} + \\
& + \frac{i}{2}(h(\delta - i\gamma^9)\chi_a)_{(p}\,(h\gamma^a(\delta - i\gamma^9))_{p')q} + \\
& + 2\mathbf{f}_{pp'}{}^{qq'}(hV)_{q'}{}^\alpha D_{\alpha 1}e^{-\Phi} - 2(h\otimes h\cdot \mathbf{f})_{pp'}{}^{qq'}V_{q'}{}^\alpha D_{\alpha 2}e^{-\Phi}\,.
\end{aligned}
\tag{D.9}
$$

Eq. (D.9) can be obtained from the $\propto e^p \wedge e^{p'}$ component of the integrability condition $D(E^{\alpha 2}V_\alpha{}^q - e^p h_p{}^q - e^a\chi_a{}^q) = 0$ of the conventional fermionic superembedding condition (3.19). A simple, but important consequence of eq. (D.9) is given by eq. (3.29),

$$
D_p h_p{}^q = -14\left((hV)_q{}^\alpha D_{\alpha 1}e^{-\Phi} + V_{q'}{}^\alpha D_{\alpha 2}e^{-\Phi}\right)\,. \tag{D.10}
$$

To derive it eqs. (C.26) and (3.36) should be taken into account.

E Derivation and complete form of some equations

E.1 Derivation of eqs. (4.11) and (4.13)

Here we present some detail on solving eq. (4.10). It is convenient to begin by contracting the indices $q_1q_2q_3$ with different sets of three projectors $(\delta \pm i\gamma^9)$. In particular, using the identity $(\delta + i\gamma^9)\gamma_b(\delta + i\gamma^9) = 0 = (\delta - i\gamma^9)\gamma_b(\delta - i\gamma^9)$ and eqs. (D.6), one finds that the contraction with three copies of the same projector produces very simple equations

$$
(\delta + i\gamma^9)_{(q_1q_2}\,((\delta + i\gamma^9)D\Upsilon)_{q_3)} = 0\,, \qquad (\delta - i\gamma^9)_{(q_1q_2}\,((\delta - i\gamma^9)D\bar{\Upsilon})_{q_3)} = 0 \tag{E.1}
$$

which imply

$$
(\delta + i\gamma^9)_{qp}D_p\Upsilon = 0\,, \qquad (\delta - i\gamma^9)_{qp}D_p\bar{\Upsilon} = 0\,. \tag{E.2}
$$

These equations are solved by

$$
D_q\Upsilon = -2i(\delta - i\gamma^9)_{qp}\tilde{\mathcal{W}}^p\,, \qquad D_q\bar{\Upsilon} = -2i(\delta + i\gamma^9)_{qp}\tilde{\mathcal{W}}^p \tag{E.3}
$$

with some fermionic field $\tilde{\mathcal{W}}^p$ whose relation the $\mathcal{W}^p$ superfields of eqs. (4.5) is to be determined.

Now, multiplying (4.10) by two (but not three as above) $(\delta+i\gamma^9)$ projectors and using eqs. (C.24), (D.3) and $(\eta-F)^{-1}=\eta+F(\eta-F)^{-1}$, after some algebra one arrives at

$$(\delta+i\gamma^9)_{q_1q_2}(\delta-i\gamma^9)_{q_3p}\left(\tilde{\mathcal{W}}^p-\mathcal{W}^p+\Upsilon\Lambda_{1p}\right)=(\gamma^b(\delta+i\gamma^9))_{q_3(q_1}(\delta+i\gamma^9)_{q_2)p}\,\mathcal{U}_b^p\ , \quad \text{(E.4)}$$

where

$$\mathcal{U}_b^p:=\mathcal{U}_b^p(\mathcal{W},F_{cd},\Upsilon)\ =(F(\eta-F)^{-1})_b{}^c(\gamma_c\mathcal{W})_p-2\Upsilon\Lambda_{1p}-\bar{\Upsilon}(h\chi_b)_p \qquad \text{(E.5)}$$

and Λ_q^1 is defined in (4.12). Notice that $\mathcal{U}_b^p:=\mathcal{U}_b^p(\mathcal{W},F_{cd},\Upsilon)$ in eq. (E.5) and, hence, the r.h.s. of eq. (E.4) does not depend on $\tilde{\mathcal{W}}^p$.

Let us discuss the decomposition of eq. (E.4) on the irreducible SO(1,7) representations with respect to the symmetrized pair of indices (q_1q_2). The list of symmetric 16×16 matrices is given in eq. (A.10) of appendix A. Only one of the irreducible parts, $\propto(\delta+i\gamma^9)_{q_1q_2}$ (which we denote by $\mathbf{1}$), contains $\tilde{\mathcal{W}}^p$ and can be used to determine its form, which we are going to discuss below. Other parts do not contain that and, hence, can (and really do) put restrictions on $\mathcal{W}$, F_{ab} and Υ.

Indeed, although one can see that the $\bar{\mathbf{1}}$, $\mathbf{8}$ and $\mathbf{56}$ irreducible parts of eq. (E.4) are satisfied identically,[24] the $\mathbf{70}$ irreducible part implies

$$\gamma^b\gamma^{a_1a_2a_3a_4}(\delta+i\gamma^9)\mathcal{U}_b=0\ . \qquad \text{(E.6)}$$

Eq. (E.6) has only trivial solutions,

$$(\delta+i\gamma^9)\mathcal{U}_b=0\ . \qquad \text{(E.7)}$$

Taking into account the explicit form of $\mathcal{U}_b$, eq. (E.5), one sees that (E.7) coincides with eq. (4.13).

Now, eq. (E.4) with $\mathcal{U}_b=0$ gives $\tilde{\mathcal{W}}^p=\mathcal{W}^p-\Upsilon\Lambda_p^1$, so that eq. (4.11) is valid,

$$D_q\Upsilon=-2i(\delta-i\gamma^9)_{qp}\left(\mathcal{W}^p-\Upsilon\Lambda_p^1\right)\ . \qquad \text{(E.8)}$$

E.2 More complete form of eqs. (4.20) and (4.26)

A more complete form of eq. (4.20) is given by

$$\begin{aligned}
D_p\mathcal{W}^{q'}=&\ ia_{ab}\gamma^{ab}_{pq'}+i\tilde{a}_{ab}(\gamma^{ab}\gamma^9)_{pq'}-\\
&-\frac{1}{2}(\gamma_a(\delta-i\gamma^9))_{pq'}(\eta-F)^{-1\,ab}D_b\Upsilon-\frac{1}{2}(\gamma_a(\delta+i\gamma^9))_{pq'}(\eta-F)^{-1\,ab}D_b\bar{\Upsilon}+\\
&+\frac{i}{16}(\gamma_a(\delta-i\gamma^9))_{pq'}\,(\mathcal{W}\gamma^b\gamma^a(\delta-i\gamma^9)h\chi_b)+\frac{i}{16}(\gamma_a(\delta+i\gamma^9))_{pq'}\,(\mathcal{W}\gamma^b\gamma^a(\delta+i\gamma^9)h\chi_b)+\\
&+\frac{i}{96}(\gamma_{abc}(\delta-i\gamma^9))_{pq'}\,(\mathcal{W}\gamma^d\gamma^{abc}(\delta-i\gamma^9)h\chi_b)+\\
&+\frac{i}{96}(\gamma_{abc}(\delta+i\gamma^9))_{pq'}\,(\mathcal{W}\gamma^d\gamma^{abc}(\delta+i\gamma^9)h\chi_b)+\\
&+\mathcal{O}(\Lambda_1)+\mathcal{O}(D_p\Lambda_1)\ .
\end{aligned} \qquad \text{(E.9)}$$

[24] The proof is basically reduced to the observations that $\gamma^a(\delta+i\gamma^9)=(\delta-i\gamma^9)\gamma^a$, $(\gamma^{abc}\gamma^9)(\delta+i\gamma^9)=(\delta-i\gamma^9)(\gamma^{abc}\gamma^9)$ and $(\delta\pm i\gamma^9)(\delta\mp i\gamma^9)=0$.

The terms denoted by $\mathcal{O}(\Lambda_1\,, D_p\Lambda_1)$ contain contributions of the fermionic flux (pull-back of the background fermionic superfield) and of the second derivative of the dilaton superfield, which is expressed through bosonic fluxes. These terms vanish in the case of flat tangent type IIB superspace.

A more complete form of eq. (4.26) reads

$$\begin{aligned}\eta_{b[c_1}\tilde{a}_{c_2c_3]} = & -\frac{\tilde{q}'}{\sqrt{|\eta+F|}}(\eta-F)_{b[c_1}F_{c_2c_3]} - \frac{1}{2\cdot 4!}(\mathcal{W}\gamma_a\gamma_{c_1c_2c_3}\gamma^9 h\gamma^a\chi_b) - \\ & -\frac{1}{4\cdot 4!}(\gamma_{c_1c_2c_3}\gamma^9)_{pq}T_{b\,p}{}^{q'}\left((\delta+i\gamma^9)_{q'q}\Upsilon-(\delta-i\gamma^9)_{q'q}\bar{\Upsilon}\right)+ \\ & +\frac{i}{8\cdot 4!}(\gamma_{c_1c_2c_3}\gamma^9)_{pq}T_{pq}{}^{q'}(\gamma_b\mathcal{W})_{q'}+\mathcal{O}(\Lambda_1)+\mathcal{O}(D_p\Lambda_1)\,.\end{aligned} \tag{E.10}$$

Using the explicit expressions for $T_{pq}{}^{q'}$ and $T_{b\,p}{}^{q'}$ in eq. (D.3) and (D.4) one can check that the third and the fourth terms in the r.h.s. of this equation are equal to zero in the case of vanishing background fluxes, in particular, in flat target superspace.

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